МРНТИ 27.31.21
УДК 517.925

https://doi.org/10.51889/5410.2022.59.66.006

BLOW-UP OF SOLUTIONS OF THE INTEGRO-DIFFERENTIAL KELVIN-VOIGHT EQUATION

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Abstract

In this work, an initial boundary value problem for a nonlinear (but without convective term) integro-differential Kelvin-Voigt equation modified with a p-Laplacian and a nonlinear source term is investigated. The integral term in the system with a convolution is called a memory term, and it indicated the viscoelastic properties of fluids. Such system of equations is called the Oskolkov equations in some papers, and it describes the motion of incompressible viscoelastic non-Newtonian fluids. In generally, there is not unique methods to prove the existence global in time of solutions to nonlinear initial-boundary value problems. However, one can response to such type questions by establishing some qualitative properties of solutions as blow up and localization in a finite time, large time behavior, and et al. In this paper, the global in time non-existence of weak generalized solutions to the studying initial boundary value problem for nonlinear modified integro-differential Kelvin-Voigt equations is proved by establishing the blowing up in a finite time property. The blow up of weak solutions to the investigating problem is obtained by using the Kalantarov-Ladyzhenskaya lemma.

Keywords: integro-differential Kelvin-Voigt equation, weak solution, blow up.

Аңдатпа

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ИНТЕГРО-ДИФФЕРЕНЦИАЛЬДЫҚ КЕЛЬФИН-ФОЙГТ ТЕҢДЕУІНІҢ ӘЛСІЗ ШЕШІМІНІҢ ШЕКСІЗДІККЕ ҰМТЫЛУЫ

Бұл жұмыста модифициаланған p-Лапласианды және сызықты емес жылу көзді интегродифференциалдық Кельвин-Фойгт теңдеулер (бірақ, конвективті мүшесі жоқ) жүйесіне қойылған бастапқы-шеттік есеп зерттелінеді. Үйірткі түріндегі интегралдық мүше жинақтау жады мүшесі деп аталады және сұйықтықтардың тұтқырлық қасиеттерін сипаттайды. Бұл теңдеулер жүйесі кейбір авторлардың жұмыстарында Осколков теңдеулер жүйесі деп те аталады және ол сығылмайтын тұтқыр ньютондық емес сұйықтықтардың қозғалысын сипаттайды. Бүгінгі таңда сызыңты емес бастапқы-шеттік есептердің шешімінің уақыт бойынша глобалды бар болуын дәлелдеу үшін біртұтас әдістер табыла қойған жоқ. Дегенмен де, шешімнің ақырлы уақытта шексіздікке ұмтылуы, локализация және үлкен уақыт тәртібі секілді сапалық қасиеттері туралы ақпарарт алуға болады. Бұл жұмыста модифициаланған p-Лапласианды және сызықты емес жылу көзді интегро-дифференциалдық Кельвин-Фойгт теңдеуіне қойылған бастапқы-шеттік есептің жалпылама әлсіз шешімінің ақырлы уақытта шексіздікке ұмтылуы қасиетін көрсету арқылы уақыт бойынша глобалды шешілмейтіндігі дәлелденді. Модифициаланған p-Лапласианды және сызықты емес жылу көзді интегродифференциалдық Кельвин-Фойгт теңдеуіне қойылған бастапқы-шеттік есептің жалпылама әлсіз шешімінің ақырлы уақытта шексіздікке ұмтылуы қасиетін көрсету КалантаровЛадыженская леммасы арқылы дәлелденді.

Түйін сөздер: интегро-дифференциалдық Кельфин-Фойгт теңдеуі, әлсіз шешім, шешімнің шексіздікке ұмтылуы.

Аннотация

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РАЗРУШЕНИЕ РЕШЕНИЯ ИНТЕГРО-ДИФФЕРЕНЦИАЛЬНОГО УРАВНЕНИЯ КЕЛЬВИНА-ФОЙТА

В работе исследуется начально-краевая задача для системы нелинейного (но без конвективного члена) интегро-дифференциального уравнения Кельвина-Фойгта, модифицированного с p-лапласианом и нелинейном источником. Интегральный член в системе в виде сверткой называется членом памяти и он указывает вязкоупругие свойства жидкостей. Данная система уравнений в некоторых работах называется также системой уравнений Осколкова, и она описывает движение несжимаемой вязкоупругой неньютоновской жидкости. В настоящей время, не существует единые методы доказательства существования решений глобально во времени нелинейных начально-краевых задач. Однако можно ответить на такие вопросы, устанавливая некоторых качественных свойств решений, такие как разрушение и локализация за конечное время, поведение при больших временах и т.д. В данной работе, путем установления свойства разрушения за конечное время, доказано глобальное во времени не существование слабых обобщенных решений исследуемой начально-краевой задачи

для нелинейного модифицированного интегро-дифференциального уравнения Кельвина-Фойгта, модифицированного с p-лапласианом и нелинейном источником. А свойства разрушения слабых решений данной исследуемой начально-краевой задачи для нелинейного модифицированного интегро-дифференциального уравнения Кельвина-Фойгта доказано с помощью леммы Калантарова-Ладыженской.

Ключевые слова: интегро-дифференциальное уравнение Кельвина-Фойгта, слабое решение, разрушения решения

1 Introduction

Let Ω be a bounded domain in $R^d, d \geq 2,$ with a smooth boundary $\partial\Omega$, and $Q_T = \{(x,t) : x \in \Omega,\ 0 < t < T\}$ is a cylinder with lateral Γ_T . In this paper, we study the following initial boundary value problem of determining the pair of functions $(u(x,t), \pi(x,t))$, which satisfy the following integro-differential Kelvin-Voigt equations modified with $p-$ Laplacian diffusion and nonlinear source term

$$u_t - div\left(\kappa\nabla u_t + \nu\,|\nabla u|^{p-2}\,\nabla u\right) - \nabla\pi - \int_0^t e^{-(t-\tau)}\Delta u(x,\tau)d\tau = \gamma\,|u|^{m-2}\,u \quad in \quad Q_{T,} \tag{1}$$

$$divu(x,t) = 0 \ \ in \ \ Q_T, \tag{2}$$

the initial condition

$$u(x,t) = u_0(x) \ \ in \ \ \Omega, \tag{3}$$

and the Dirihlet boundary condition

$$u(x,t) = 0 \ \ on \ \ \Gamma_T. \tag{4}$$

Here $u(x,t)$ is the velocity field, $\pi(x,t)$ is the pressure, $u_0(x)$ is the given initial velocity. The constant ν accounts for the dynamic viscosity, whereas κ is a length scale parameter characterizing the fluid elasticity in the sense that the ratio $\frac{\kappa}{\nu}$ is a relaxation time scale, i.e. a characteristic time required for a viscoelastic fluid to relax from a deformed state to its equilibrium configuration. The exponents p and m are given positive numbers, such that

$$1 < p, m < \infty \tag{5}$$

The system (1)-(2), in the case $\gamma = 0$ and $p = 2$, is called integro-differential Kelvin-Voigt or Oskolkov system and describes the motion of viscoelastic incompressible non-Newtonian fluids, that needs time to start motion after the force is applied [1], and the correctness of various initial boundary value problems for them studied in some works in [2]- [3], and et.al. Karazaeva [4]- [5], Ziviagin-Turbin [1], Yushkov [6]. The integral term of (1) with the convolution kernel $K(t) = e^t$ is called a memory term, and it designs the viscoelastic property of non-Newtonian fluids [1]. The existence, uniqueness and some qualalive properties such as blow up in a finite, and large time behavior of weak solutions of the initial-boundary value problem for modified Kelvin-Voigt equations with p−Laplacian and damping term (without memory term) have been investigated by some authors, for instance, see [7], [8]- [12]. The existence, uniqueness and blows up of weak solutions for the initial-boundary value problem for integro-differential Kelvin-Voight equations with cubie source term, i.e. in the case p = 2 and m = 2 were established by Yushkov in [6].

2 Preliminaries

In this section, we introduce some auxiliary lemmas and functional spaces that will be used throughout the paper. For the definitions, notations of the function spaces and for their properties, we address the reader also to the monographs [13,14]. In this wirk, we establish the results of blow up in a finite time of solutions for more generalized integro differential Kelvin-Voight equations, which the some results of [6] are valid in particulat cases, of $p = 2$ and $m = 2$.

2.1 Functional spaces

In this section, we introduce the necessary notation, the definition of weak solution to problem (1)-(4), and an important lemmas which are used below in this paper. We define the functional spaces used throughout the paper and describe briefly their properties. Namely, the norm of $L_p(\Omega)$ is denoted by $\|u\|_p = \left(\int_\Omega |u|^p \, dx\right)^{\frac{1}{p}}$. Let us also introduce the following functional spaces widely used in the Mathematical Fluid Mechanics:

$$V(\Omega) := \{u \in C^\infty(\Omega):\ divu = 0 \ \ and \ \ u = 0 \ \ on \ \ \partial\Omega\}; \tag{6}$$

$$H(\Omega) := \overline{V(\Omega)}^{\|u\|_2}; \tag{7}$$

$$\mathrm{X}(\Omega) := \overline{V(\Omega)}^{\|u\|_{H^1}}; \tag{8}$$

$$X_p(\Omega) := \overline{V(\Omega)}^{\|u\|_{W_p^1}}; \tag{9}$$

The blowing up of a solutions will be established by using the following lemma, which the proof can be found in [15].

Lemma 1. Suppose that, for $t > 0$, a positive, twice-differentiable function $\Psi(t)$ satisfies the inequality:

$$\Psi''(t)\Psi(t) - (1+\alpha)\left(\Psi'(t)\right)^2 \ge -2B_1\Psi'(t)\Psi(t) - B_2\Psi^2(t), \tag{10}$$

where $\alpha > 0,\ B_1, B_2 \ge 0$. If $\Psi(0) > 0,\ \ \Psi'(0) > -\gamma_2\alpha^{-1}\Psi(0)$ and $B_1 + B_2 > 0$, then $\Psi(t) \to +\infty$ as

$$t \to t_1 \le \frac{1}{2\sqrt{B_1^2 + \alpha B_2}} \ln \frac{\gamma_1\Psi(0) + \alpha\Psi'(0)}{\gamma_2\Psi(0) + \alpha\Psi'(0)}. \tag{11}$$

Here $\gamma_1 = -B_1 + \sqrt{B_1^2 + \alpha B_2},\ \ \gamma_2 = -B_1 - \sqrt{B_1^2 + \alpha B_2}$.

We now define a weak solution to the problem (1)-(4) as following sense, which we work with in this paper

Definition 1. A function $u(x,t)$ is a weak solution to the problem (1)-(4), if:

1. $L^\infty(0,T; X(\Omega)) \cap L^p(0,T; X_p(\Omega)) \cap L^m(Q_T)$;
2. $u(x,0) = u_0(x)$ a.e. in Ω;
3. For every $\varphi \in X(\Omega) \cap X_p(\Omega) \cap L^m(\Omega)$ and for a.a. $t \in (0,T)$ holds

$$\frac{d}{dt}\left(\int_\Omega u\varphi dx + \kappa\int_\Omega \nabla u \nabla\varphi dx\right) + \nu\int_\Omega |\nabla u|^{p-2} \nabla u \nabla\varphi dx = \gamma\int_\Omega |u|^{m-2} u\varphi dx - \int_0^t e^{-(t-\tau)} d\tau \int_\Omega \nabla u(\tau)\nabla\varphi dx. \tag{12}$$

3 Main result. In this section we establish the following main result.

Theorem 1. Suppose that $u_0(x) \in H$ and fulfilled is next condition

$$m \ge p,\ \ p \le 2,$$

the following condition is satisfied

$$\gamma\|u\|_m^m - \nu\|\nabla u\|_p^p > \sqrt{\frac{\delta}{\gamma - 1}}\left(\frac{1}{2}\|u\|_2^2 + \frac{\kappa}{2}\|\nabla u\|_2^2 + C_0\right).$$

Then there exists a finite time $T_{\max} < \infty$ such that a weak solution to problem (1)-(4) blows up, i.e

$$\frac{1}{2}\|u\|_2^2+\frac{\kappa}{2}\|\nabla u\|_2^2 \to \infty \text{ as } t\to T_{\max}.$$

where $\delta := m\beta;\ \chi := \frac{m\alpha}{2};\ \alpha := \frac{1}{e^{\varepsilon}};$

$$\beta := \frac{\nu^2(m-p)^2(p-1)}{m^2\kappa^2}\frac{e^{\varepsilon}}{e^{\varepsilon}-1}\varepsilon^{\frac{1}{p-1}}+\frac{e^{\varepsilon}-1}{e^{\varepsilon}}+\frac{2}{m\kappa}+\frac{(m-1)^2+1}{m^2\kappa^2}\frac{2e^{\varepsilon}}{e^{\varepsilon}-1};\ \varepsilon\in\left(0,\ln\frac{m}{2}\right).$$

Proof. Let us define the energy function

$$\Psi(t) := \frac{1}{2}\|u\|_2^2+\frac{\kappa}{2}\|\nabla u\|_2^2+C_0,$$

where C_0 is positive constant, that can be choosen below.

Let us multiply the equation (1) by u and u_t, and integrate by x over Ω. After integrating by parts, we have the following equalities,

$$\Psi'(t)+\nu\|\nabla u\|_p^p+\int_0^t e^{-(t-\tau)}\int_\Omega \nabla u(\tau)\nabla u(t)dxd\tau=\gamma\|u\|_m^m \quad , \tag{13}$$

$$\|u_t\|_2^2+\kappa\|\nabla u_t\|_2^2+\frac{\nu}{p}\frac{d}{dt}\|\nabla u\|_p^p+\int_0^t e^{-(t-\tau)}\int_\Omega \nabla u(\tau)\nabla u_t(t)dxd\tau=\frac{\gamma}{m}\frac{d}{dt}\|u\|_m^m, \tag{14}$$

respectively.

Next, substituting the expression for $\gamma\|u\|_m^m$ from (13) into (14), we obtain the following equality

$$\begin{gathered}\|u_t\|_2^2+\kappa\|\nabla u_t\|_2^2+\frac{\nu}{p}\frac{d}{dt}\|\nabla u\|_p^p+\int_0^t e^{-(t-\tau)}\int_\Omega \nabla u(\tau)\nabla u_t(t)dxd\tau=\\ \frac{1}{m}\Psi''(t)+\frac{\nu}{m}\frac{d}{dt}\|\nabla u\|_p^p+\frac{1}{m}\frac{d}{dt}\int_0^t e^{-(t-\tau)}\int_\Omega \nabla u(\tau)\nabla u(t)dxd\tau,\end{gathered} \tag{15}$$

let us calculate the derivative of the last term on the right hand side of (15):

$$\begin{gathered}\frac{d}{dt}\int_0^t e^{-(t-\tau)}\int_\Omega \nabla u(\tau)\nabla u(t)dxd\tau=\|\nabla u\|_2^2+\\ \int_0^t e^{-(t-\tau)}\int_\Omega \nabla u(\tau)\nabla u_t(t)dxd\tau-\int_0^t e^{-(t-\tau)}\int_\Omega \nabla u(\tau)\nabla u(t)dxd\tau.\end{gathered} \tag{16}$$

Plugging (16) into (15), we obtain

$$\begin{gathered}\|u_t\|_2^2+\kappa\|\nabla u_t\|_2^2=-\frac{\nu(m-p)}{mp}\frac{d}{dt}\|\nabla u\|_p^p+\frac{1}{m}\Psi''(t)+\frac{1}{m}\|\nabla u\|_2^2-\\ \frac{1}{m}\int_0^t e^{-(t-\tau)}\int_\Omega \nabla u(\tau)\nabla u(t)dxd\tau-\frac{m-1}{m}\int_0^t e^{-(t-\tau)}\int_\Omega \nabla u(\tau)\nabla u_t(t)dxd\tau.\end{gathered} \tag{17}$$

Next, we estimate the each term on the right hand side of (17). For the first term we using the assumption $1<p\le 2$, we have

$$\left|-\frac{d}{dt}\|\nabla u\|_p^p\right| \le p\int_\Omega |\nabla u|^{p-1}\ \nabla u_t dx \le \frac{p}{2\varepsilon_0}\int_\Omega |\nabla u|^{p-2}\ dx + \frac{p\varepsilon_0}{2}\int_\Omega |\nabla u_t|^2\ dx \le \frac{p\varepsilon_0}{2}\|\nabla u_t\|_2^2 +$$
$$\frac{p(p-1)}{2\varepsilon_0}\varepsilon^{\frac{1}{p-1}}\|\nabla u\|_2^2 + \frac{p(2-p)}{2\varepsilon_0}\frac{|\Omega|}{\varepsilon^{\frac{1}{2-p}}} \le \frac{p\varepsilon_0}{2}\|\nabla u_t\|_2^2 + \frac{p(p-1)}{\kappa\varepsilon_0}\varepsilon^{\frac{1}{p-1}}\Psi(t) + \frac{p(2-p)}{2\varepsilon_0}\frac{|\Omega|}{\varepsilon^{\frac{1}{2-p}}}, \tag{18}$$

where ε_0 and ε are an arbitrary positive constants, which we choose below.

Using the Holder and Young inequalities we estimate fourth and fifth terms on the right hand side of (17).

$$\left|-\int_0^t e^{-(t-\tau)}\int_\Omega \nabla u(\tau)\nabla u(t)dxd\tau\right| \le \frac{1}{\varepsilon_1\kappa}\Psi(t) + \frac{\varepsilon_1}{\kappa}\int_0^t e^{-(t-\tau)}\Psi(\tau)d\tau, \tag{19}$$

$$\left|-\int_0^t e^{-(t-\tau)}\int_\Omega \nabla u(\tau)\nabla u(t)dxd\tau\right| \le \frac{\varepsilon_2}{2}\|\nabla u_t\|_2^2 + \frac{1}{\varepsilon_2\kappa}\int_0^t e^{-(t-\tau)}\Psi(\tau)d\tau. \tag{20}$$

Plugging (18), (19) and (20) into (17), we have

$$\left(1-\frac{(m-1)\varepsilon_2}{m\kappa}-\frac{\nu(m-p)\varepsilon_0}{2m\kappa}\right)\left(\|u_t\|_2^2+\kappa\|\nabla u_t\|_2^2\right) \le \frac{1}{m}\Psi''(t) + \frac{2}{m\kappa}\left(\frac{1}{2}\|u\|_2^2+\frac{\kappa}{2}\|\nabla u\|_2^2 + C_0\right)$$
$$\left(\frac{\nu(m-p)(p-1)}{m\kappa\varepsilon_0}\varepsilon^{\frac{1}{p-1}}+\frac{1}{m\kappa\varepsilon_1}\right)\Psi(t) + \frac{\varepsilon_1\varepsilon_2+m-1}{\varepsilon_2 m\kappa}\int_0^t e^{-(t-\tau)}\Psi(\tau)d\tau \le \frac{1}{m}\Psi''(t) + \tag{21}$$
$$\left(\frac{\nu(m-p)(p-1)}{m\kappa\varepsilon_0}\varepsilon^{\frac{1}{p-1}}+\frac{1}{m\kappa\varepsilon_1}+\frac{2}{m\kappa}\right)\Psi(t) + \frac{\varepsilon_1\varepsilon_2+m-1}{\varepsilon_2 m\kappa}\int_0^t e^{-(t-\tau)}\Psi(\tau)d\tau,$$

where above an arbitrary constant C_0 has been chosen as

$$C_0 := \frac{\nu(m-p)(2-p)\kappa}{4\varepsilon_0}\frac{|\Omega|}{\varepsilon^{\frac{1}{2-p}}}.$$

If $\Psi(0)>0$, then there exists $t_1>0$ [6], such that

$$\Psi'(t)>0 \quad \text{for all } t\in[0,t_1]. \tag{22}$$

Thus, using (22), we get the following inequality:

$$\int_0^t e^{-(t-\tau)}\Psi(\tau)d\tau \le \Psi(\tau)-\Psi(0)e^{-t}-\int_0^t e^{-(t-\tau)}\Psi'(\tau)d\tau \le \Psi(\tau). \tag{23}$$

Plugging (23) into (21), we get

$$\left(1-\frac{(m-1)\varepsilon_2}{m\kappa}-\frac{\nu(m-p)\varepsilon_0}{2m\kappa}\right)\left(\|u_t\|_2^2+\kappa\|\nabla u_t\|_2^2\right) \le \frac{1}{m}\Psi''(t) +$$

$$\left(\frac{\nu(m-p)(p-1)}{m\kappa\varepsilon_0}\varepsilon^{\frac{1}{p-1}}+\frac{1}{m\kappa\varepsilon_1}+\frac{2}{m\kappa}+\frac{\varepsilon_1\varepsilon_2+m-1}{\varepsilon_2 m\kappa}\right)\Psi(t) \qquad (24)$$

If we choose the arbitrary constants ε_0, ε_1 and ε_2 in (24) such that

$$\varepsilon_0=\frac{m\kappa}{\upsilon(m-p)}\frac{e^{\varepsilon}-1}{e^{\varepsilon}},\quad \varepsilon_1=\frac{1}{m\kappa}\frac{2e^{\varepsilon}}{e^{\varepsilon}-1},\quad \varepsilon_2=\frac{m\kappa}{m-1}\frac{e^{\varepsilon}-1}{2e^{\varepsilon}},$$

then we have

$$\alpha\left(\|u_t\|_2^2+\kappa\|\nabla u_t\|_2^2\right)\le\frac{1}{m}\Psi''(t)+\beta\Psi(t), \qquad (25)$$

Where

$$\alpha:=\frac{1}{e^{\varepsilon}};\ \ \beta:=\frac{\nu^2(m-p)^2(p-1)}{m^2\kappa^2}\frac{e^{\varepsilon}}{e^{\varepsilon}-1}\varepsilon^{\frac{1}{p-1}}+\frac{e^{\varepsilon}-1}{e^{\varepsilon}}+\frac{2}{m\kappa}+\frac{(m-1)^2+1}{m^2\kappa^2}\frac{2e^{\varepsilon}}{e^{\varepsilon}-1},$$

with an arbitrary $\varepsilon>0$.

Next, [8] using Holder's and Young's inequalities, we also derive the following inequality:

$$\left(\Psi'(t)\right)^2\le 2\Psi(t)\left(\|u_t\|_2^2+\kappa\|\nabla u_t\|_2^2\right). \qquad (26)$$

Now, combining (26) with (25), we get

$$\alpha\Psi(t)\left(\|u_t\|_2^2+\kappa\|\nabla u_t\|_2^2\right)\le\frac{1}{m}\Psi''(t)\Psi(t)+\beta\left(\Psi(t)\right)^2, \qquad (27)$$

which yealds

$$\frac{m\alpha}{2}\left(\Psi'(t)\right)^2\le\Psi''(t)\Psi(t)+m\beta\left(\Psi(t)\right)^2. \qquad (28)$$

If follows (28) that

$$\Psi''(t)\Psi(t)-\chi\left(\Psi'(t)\right)+\delta\left(\Psi(t)\right)^2\ge 0 \qquad (29)$$

where $\delta=m\beta$, $\chi=\frac{m\alpha}{2}$ and $\varepsilon\in\left(0,\ln\frac{m}{2}\right)$.

Hence, by the assumption of Theorem 1, the function $\Psi(t)$ satisfies all the conditions of Lemma 1 with the constants $B_1=0$, $B_2=\delta$ and $\alpha=\chi-1>0$.

Thus, by Lemma 1, the statement in Theorem 1 is true.

4 Conclusion

In the paper, the space of a weak generalized solutions of initial boundary value problem for a nonlinear (but without convective term) integro-differential Kelvin-Voigt equation modified with a p-Laplacian and a nonlinear source term is defined. Under suitable conditions on the data of the problem, the blowing up property of weak solutions in a finite time is established.

5 Acknowledgment

This research has been funded by the Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan, Grant No. AP08052425.

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