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To Memory Shaltay Smagulov

ABOUT SOLUTION OF SINGULAR BILINEAR STOCHASTIC SYSTEMS ON THE BASE SMAGULOV`S CONDITION

Abstract

This work considered class singular bilinear stochastic systems in Langevin's form. The authors presented the definition and application of apparatus, a class of pseudo-semi-inverse matrices, and, for the first time, included Sh.S. Smagulov's initial condition for the class of singular nonlinear stochastic systems in bilinear case. The article considers a system in the form Map «Input – Output» for a class with several inputs in the Langevin's form. On the base of connection between Langevin's form and Volterra's form are proved the theorem about the construction of Volterra`s model for a class of singular nonlinear stochastic systems in bilinear case. Also, authors proved theorems about uniqueness, convergence and finitely (on the base class of nilpotent matrices of S. Li) for this Volterra's model in Ito`s form in conception of describing the system in form Map «Input – Output» for the above class of systems, but with several inputs. As well known in the problem statement for solving the singular system, in other investigations, initial condition stated, in our view, is incorrect, or this condition is absent. Therefor due to Sh.S. Smagulov's initial condition and on the base apparatus R.S. Sudakov's class of pseudo-semi-inverse matrices and using Sh.L.Sobolev's annulator can build solutions above named of a class of singular nonlinear stochastic systems in bilinear case.

Keywords: singular, nonlinear, stochastic systems, bilinear, initial condition, annulator.

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О РЕШЕНИИ СИНГУЛЯРНЫХ БИЛИНЕЙНЫХ СТОХАСТИЧЕСКИХ СИСТЕМ С УСЛОВИЕМ СМАГУЛОВА

Аннотация

В данной работе рассматривается класс сингулярных билинейных стохастических систем в форме Ланжевена. Авторы представили определение и применение аппарата класса псевдо-полуобратных матриц и впервые ввели начальное условие Ш.С. Смагулова для класса сингулярных нелинейных стохастических систем в билинейном случае. В статье рассматривается система в виде отображения «Вход – Выход» для класса с несколькими входами в форме Ланжевена. На основе связи формы Ланжевена и формы Вольтерра доказывается теорема о построении модели Вольтерра для класса сингулярных нелинейных стохастических систем в билинейном случае. Также авторами доказаны теоремы об единственности, сходимости и конечности (на основе класса нильпотентных матриц С. Ли) для этой модели Вольтерра в форме Ито в концепции описания системы в отображения «Вход – Выход» для вышеупомянутого класса систем, но с несколькими входами. Как известно, в постановке задачи для решения сингулярных систем, в других исследованиях, начальное условие указано, на наш взгляд, неверно или это условие вообще отсутствует. Поэтому, благодаря начальному условию Ш.С. Смагулова и на основе аппарата псевдо-полуобратных матриц Р.С. Судакова и с применением аннулятора С.Л. Соболева можно построить решения для вышеназванного класса сингулярных нелинейных стохастических систем в билинейном случае.

Ключевые слова: сингулярные, нелинейные, стохастические системы, билинейные, начальное условие, аннулятор.

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Шалтай Смагуловты еске алу

СМАГУЛОВ ШАРТЫНДАҒЫ СИНГУЛЯРЛЫ БИСЫЗЫҚТЫ СТОХАСТИҚАЛЫҚ ЖҮЙЕЛЕРДІ ШЕШУ ТУРАЛЫ

Аңдатпа

Жұмыста Ланжевен түріндегі сингулярлы бисызықты стохастикалық жүйелер класы қарастырылған. Авторлар псевдо-жартылай кері матрицалар класының аппаратының анықтамасы мен қолданылуын келтіреді және бисызықты жағдайдағы сингулярлы сызықты емес стохастикалық жүйелер класы үшін Ш.С. Смағұловтың бастапқы шартын алғаш рет енгізді. Мақалада «Кіру – Шығу» бейнелеу түріндегі Ланжевеннің бірнеше кіру формасындағы класы үшін жүйе қарастырылады. Ланжевен мен Вольтерра формалары арасындағы байланыс негізінде бисызықты жағдайдағы сингулярлы сызықты емес стохастикалық жүйелер класы үшін Вольтерра моделінің құрылуы туралы теорема дәлелденеді. Сондай-ақ, бірнеше кірудегі жоғарыда аталған жүйелерге «Кіру – Шығу» бейнелеу сипаттау концепциясында Ито түріндегі Вольтерраның осы моделі үшін бірегейлік, жинақтылық және ақырлылық (С. Лидің нильпотентті матрицалар класы негізінде) туралы түйіндер дәлелденіп беріледі. Белгілі болғандай, басқа зерттеулерде сингулярлы жүйелерді шешу есебінің қойылуында, біздің ойымызша, қөрсетілген бастапқы шарты дұрыс емес немесе бұл шарт мүлде жоқ. Сондықтан, Ш.С. Смағұловтың бастапқы шарты арқылы, Р.С. Судаковтын псевдо-жартылай кері матрицалар аппараты негізінде және С.Л. Соболевтің аннуляторын пайдаланып, жоғарыда аталған бисызықты жағдайдағы сингулярлы сызықты емес стохастикалық жүйелер класының шешімдерін тұрғызуға болады.

Түйін сөздер: сингулярлы, сызықты емес, стохастикалық жүйелер, бисызықты, бастапқы шарты, аннуляторы.

Main provisions

In the work author stated of the initial condition of Sh.S. Smagulov in the form of a theorem. Also, in the article presented analytical solution of singular, deterministic, stochastic bilinear systems in Langevin's form based on applying a class of pseudo-semi-inverse matrices with the S.L. Sobolev's annulator used. The system's transition from Langevin's to Ito's form is based on transformation formulas. Solution of singular, deterministic, stochastic bilinear systems is constructed as Volterra's model based on the theory of Volterra's series. The research of Volterra's model is presented as an infinite series on finiteness properties based on applying a nilpotent class forming a Lie algebra. The uniqueness of the Volterra's series proved.

Introduction

The relevance of the research problem presented in this article lies in the fact that there are currently no:

1. Methods of analytical solution of classes of singular systems. However, the world solves the class mentioned above of systems by numerical methods depending on their class.

2. In many studies of classes of singular systems, the state vector is deterministic and singular systems practically lack an initial condition. However, due to the dynamism property, such an initial condition must necessarily be present. On the other hand, in many studies, the initial condition is present, but in our opinion, it is set incorrectly, due to the active action of the matrix (our operator, representing a degenerate or rectangular shape on the system, automatically generates its actions on the initial condition. The correct setting of initial conditions is crucial for the accurate analysis and control of singular systems, a point we will further elaborate on in this article.

Based on the above, this paper's research topics are very relevant.

The research presented in this article aims to develop a new analytical method for solving classes of deterministic, stochastic bilinear systems with the properties of singularity (descriptor) and nonincorrectness based on the initial condition of Sh.S. Smagulov, apparatus of pseudo–semi–inverse matrices, and theory of Volterra's series.

Research methodology

The problem of analytically solving singular, deterministic, stochastic billing systems rests on the solution of the following tasks.

1. The initial condition setting for this class of systems needs to be more correct.

2. Properties of the singularity of the system automatically determine properties of the incorrectness of this system.

In connection with the above, we can state that the methodology of solving problems formulated in this work is divided into two stages. The first stage eliminates the disadvantages of setting the initial condition, which is generally absent, or the authors ignore the properties of the systems dynamism. The second stage of the study consists of the following sequences of steps.

Step 1. Analytical solution of singular, deterministic, stochastic bilinear systems, Langevin's formula presented.

Step 2. The system transition mentioned above from Langevin's form to Ito's form.

Step 3. Constructing solutions of singular, deterministic, stochastic bilinear systems in the form of Volterra's model.

Step 4. A study of Volterra's model presented as an infinite series on finiteness properties.

Step 5. A study of Volterra's model presented as an infinite series on uniqueness properties.

Results of the study

As previously noted, the methodology for solving the problems formulated in this work is divided into two stages.

In the first stage, the initial condition of Sh.S. Smagulov formulated in the form of a theorem of the same name. In the second stage, at the first step, the analytical solution of singular, deterministic, stochastic bilinear systems presented in Langevin's form based on applying a class of pseudo-semi-inverse matrices with the S.L. Sobolev's annular used. The system's transition from Langevin's to Ito's form in the second step is based on transformation formulas. In the third step, the solution of singular, deterministic, stochastic bilinear systems is constructed as Volterra's model based on the theory of Volterra's series. In the fourth step, the study of Volterra's model is presented as an infinite series on finiteness properties based on applying a nilpotent class forming a Lie algebra. In the fifth step, the uniqueness of the Volterra's series proved.

The device pseudo inverse Moore – Penrose's matrices were presented for the first time in [1]. In [2], Sobolev's matrices are given, and S.L. Sobolev's theorem for the solution of systems of algebraic equations, which was also used in [1]. In [3], the theory of the class mentioned above of nonlinear pseudo–semi–inverse matrices for the first time presented matrices of R.S. Sudakov and her application to problems of an assessment of the reliability of systems. In [3], the remark is made that pseudo the return matrices; it is difficult to use the theory in several tasks from linear algebra.

The definition of Drazin matrices in [4] introduced a unique class of matrices, characterized by their nilpotent structure. Their application to the solution of nonlinear singular differential equations, using the theorem of S. L. Sobolev, was also discussed. It's important to note that the Drazin matrices form a distinct and narrow class with properties that are not commutative, as mentioned in [3] for pseudo-inverse matrices.

Now, we present an analysis of Stochastic Realization Methods. From it [4,5] came a great idea "to formulate the realization theory of stochastic processes in the same natural way as it is done in the case of deterministic systems. "In this direction, one of the most profound studies of linear discrete stochastic implementation problems was done in [6]. The first stage of the algorithm for solving this problem is to solve the well-known problem of linear deterministic implementation [7] for the case when parameters are set in the "weighting function". In the second step, we solve the problem of spectral factorization, which is completed by solving the algebraic Riccati equation. You can specify a number of papers that have used this idea [8,9]; however, in the studies mentioned above.

In [10], the control problem of switched singular systems was investigated, with the aim of compressing their inconsistent state jumps when a switch occurs between two different singular subsystems. In [11], a definition of a transform was presented, which reformulates a system with delays into a singular linear system of differential equations. This transform is significant as it introduces non-square constant matrices with a greater number of columns than rows. In engineering applications, the complexity increase implies that the accuracy of these systems cannot be adequately described by linear singular systems, posing a significant challenge that this research aims to address.

Proving the results valid for complex-valued functions could be considered to be a future problem. Moreover, as done in [11], we can consider analyzing a system with delay by reformulating it into a singular linear system of differential equations as future work. We believe that the results of this paper are of great significance for the relevant community and can be used, for instance, to investigate switched singular time-delay systems. The authors explored the singular system within a category of DEs with multiple delays in his work documented in [11].

It [12] was developed and summarizes the results of previous studies for non-stationary time-continuous linear stochastic systems. The focus is on the probabilistic aspects, the conclusion of the natural representation of the process state as the state of the stochastic limited "input-output" display, and the implementation as a "representation of the update release".

Imposed on the process conditions, it may be time-dependent and can lead to realization of larger dimensions. Classes of fixed order models ("guaranteed models") were defined as having a common correlation matrix of combined vector's outputs in the range of determination greater than or equal to the correlation matrix of the process. This approach is not just theoretical, but also applied in the practical problem of detecting the smallest order models of high-order correlation matrices in numerical form, making it highly relevant to real-world problems.

Wiener's theory was used to prove the theorem of stochastic nonlinear realization existence in Hilbert's space. Its analysis critically depends on the assumption that Volterra's kernels are set to update the representation of the output process, as the problem of definition of these kernels is non-trivial.

It [13] presented the initial condition for the Aver-Stokes equation. These ideas were developed further [14-18].

Discussion

The basic definitions of the theory of pseudo- semi-inverse matrices.

Definition 1. For the matrix E of dimension *m* by *n*, the matrix
E^c is called a pseudo semi-inverse matrix if the following relationship

$$E = EE^cE, \tag{1}$$

Definition 2. Matrix R is called left S. L. Sobolev's annulator, if
$R = I - EE^c$
In presented algorithms for finding the pseudo semi-inverse matrices, but not the only way.

Mathematical formulation of the problem

Consider the Singular Bilinear Stochastic System of the Langevin's form

$$\begin{gathered} Ex(t) = Ax(t) + Bx(t)U(t) \\ Ex(t_0) = Ex_0 \end{gathered} \tag{2}$$

$$y(t) = Vx(t) \tag{3}$$

where A, B – square matrices, dimensional m x n , x(t) –n-dimensional vector-column , C- the row of identical dimensionalities, U(t) – standard Wiener process. For system (2)- (3) required to find the Map "Input-Output". Input U(t) and Output Y(t) can be measured at any moment of time.

$$\det(E) = 0.$$

It is supposed, that processes x(t) is independent for all t, t_0, t_1 and the matrix $Q = Ex_0x_0^T$

Sh.S. Smagulov's Theorem 1 Initial condition for class of singular deterministic and stochastics systems presented by formula (2).

Proof: We use properties of action singular operator on system.

Theorem 1 is proved.

Definition 3. Condition (2) called Sh.S. Smagulov's initial condition for singular systems.

Theorem 2. For Singular Bilinear Stochastic Systems (2) – (3) with n there exists mathematical model by following form:

$$x(t) = A_1\tilde{x}(t) + B_1\tilde{x}U(t) + B_2V(\xi) \tag{4}$$

Where

$$A_1 = E^c \cdot A, \quad B_1 = E^c \cdot B, \quad B_2 = I - E^c \cdot E \tag{5}$$

E^c – Pseudo-semi-inverse matrix;

$V(\xi)$ – constant vector,

If and only if

$$E \cdot E^c\big(Ax(t) + B_1xU(t)\big) = Ax(t) + BxU(t) \tag{6}$$

Proof:

Suppose, that (4) is true. Multiplying of both parts of equation (4) in left on matrix E, we have

$$Ex(t) = E\big(A_1x(t) + B_1x(t)U(t)\big) + E(I - E^cE)$$

or

$$Ex(t) = EE^c\big(Ax(t) + Bx(t)U(t)\big) + E - EE^cE$$

Since

$$E = EE^cE$$

According to condition (6), we obtain following equation

$$Ex(t) = Ax(t) + Bx(t)U(t)$$

Theorem 2 is proved.

On the basis of Theorem 2, we use the representation of the system (2) as (4).

Problem state

Required to find the solution of system (2) - (3) in Langevin's form with several inputs due to the transformation we can receive in the form of Volterra's stochastic series in Ito's form or receive Map «input-output» and investigate its uniqueness, convergence and finite properties.4

Central to our analysis are one-dimensional centered random variables, which we consider as the norm for convergence in our study.

$\|\xi\| = \sqrt{E\xi^2}$ The convergence of n-dimensional variables

$$\eta = (\eta_1, \dots, \eta_n)$$

is defined by norm

$$\|\eta\| = \sqrt{\sum_{i=1}^{m} \eta_i^2}$$

norm of matrix random variables

$$\xi = (\xi_{ij})$$

is defined as operaturny norm of the type:

$$\|\xi\| = sup\ g \neq 0\ \frac{\|Ig\|}{g}\ .$$

Where

$$I = (\|\xi_{ij}\|)$$

On the basis of Theorem 1, we use the representation of the system (2) as (4).

Problem state

It is required to find the solution of system (4) - (3) in the form of Volterra's stochastic series or receive Map «input-output» and investigate its uniqueness, convergence and finite properties.

The solution to the problem is not just a mere result but a profound revelation, as it follows from the following theorem.

Theorem 3. For the stochastic bilinear system (2) - (3) in Langevin's form with several inputs there is only unique solution presented in the form of Volterra`s series: in Ito`s form

$$y(t) = ce^{At}x_0 + \sum_{i=1}^{\infty}\sum_{j=1}^{m} \dots \sum_{p=1}^{m} \int_0^t \int_0^{e_1} \int_0^{e_{i-1}} ce^{A(t-e_1)} B_j e^{A(e_1-e_2)} B_p e^{A(e_{i-1}-e_1)} \quad (7)$$

$$x_0 dW_j(e_1) \dots dW_p(e_1) + \sum_{i=1}^{\infty} \sum_{j=1}^{m} \int_0^t \int_0^{e_1} \int_0^{e_{i-1}} ce^{A(t-e)} B_j e^{A(e_1-e_2)}$$

$$B_j \dots e^{A(e_{i-1}-e_1)} \cdot x_0 dW_j(e_1) \dots dW_j(e_1)$$

which uniformly converges under a sufficient condition: for any matrices $A, B_1 (i = 1, \dots, m), x_0$ and any fixed t, exists $\varepsilon > 0$,

and that at any $B_1 (i = 1, \dots . m)$:

$$\|B_i\| < \varepsilon, (i = 1, \dots, m), \quad (8)$$

which is finite, if the set of matrices is nilpotent (Lie`s algebra).

Proof analogously as proof for deterministic bilinear systems (See [14]).

Proof: We consider system in form Map «Input – Output» for class of systems, but with several inputs in the Langevin's form. Using connection between Langevin's form and Volterra's form. (See [14]).

The construction of Volterra's model.

Applying Ito's formula of stochastic differentiation we have:

$$dd\left(e^{-At}x(t)\right) = e^{-At}dt - Ae^{-At}x(t)dt = e^{-At}(dx - Axdt) = e^{-A(t)}\left(\sum_{i=1}^{N} B_i x(t) dW_i(t)\right) \quad (9)$$

Integrating both parts, we obtain:

$$e^{-At}x(t) - x_0 = \sum_{i=1}^{N} \int_0^t e^{-Ae} B_i x(e) dW_i(e) \quad (10)$$

From here

$$x(t) = e^{-At}x_0 + \sum_{i=1}^{N}\int_0^t e^{A(t-e)}B_i x(e)dW_i(e) \tag{11}$$

The right-hand part of the integral equation (10) denoted through $\hat{G}$ where the operator $\hat{G}$ – acts from the space of all Gaussian processes with the norm $sup\|x(\varepsilon)\|$ on the interval (0,t) in himself.

For the solution of the equation (11) we write it in a recurrent form:

$$x^{(k+1)}(t) = e^{At}x_0 + \sum_{i=1}^{N}\int_0^t e^{A(t-e)}B_i x^{(k)}(e)dW_i(e) \tag{12}$$

To the equation (12) we apply the method of consecutive approximation, and then we obtain:

$$x^{(1)}(t) = e^{At}x_0 + \sum_{i=1}^{N}\int_0^t e^{A(t-e)}B_i x_0 dW_i(e)$$

$$x^{(2)}(t) = e^{At}x_0 + \sum_{i=1}^{N}\int_0^t e^{A(t-e)}B_i x^{(1)}(e)dW_i(e) = e^{At}x_0 + \sum_{i=1}^{N}\int_0^t e^{A(t-e)}B_i e^{Ae_1}x_0 dW_i(e)$$
$$+\sum_{i=1}^{\infty}\sum_{j=1}^{N}\int_0^t\int_0^e\int_0^e e^{A(t-e_1)}B_j e^{A(e_1-e_2)}\cdot B_j x_0 dW_i(e_2)dW_i(e_1)$$

Finally directing

$k \to \infty$, we have:

$$x(t) = ce^{At}x_0 + \sum_{i=1}^{\infty}\sum_{j=1}^{N}\dots\sum_{p=1}^{N}\int_0^t\int_0^{t_1}\dots\int_0^{t_{i-1}} e^{A(t-e_1)}B_j e^{A(e_1-e_2)} \tag{13}$$

$j \neq \cdots \neq p$

$$B_p e^{A(e_{i-1}-e_j)}x_0 dW_i(e_1)\dots dW_p(e_p) + \cdots$$
$$+\sum_{i=1}^{\infty}\sum_{j=1}^{N}\int_0^t\int_0^{e_1}\int_0^{e_{i-1}} e^{A(t-e_1)}B_i e^{A(e_1-e_2)}B_i \dots e^{A(e_{i-1}-e_i)}x_0 dW_j(e_1)\dots dW_j(e_i)$$

From here we have (8).

Notation. Representation (13) contains from real and stochastic of parts. However, in this case, members obtained by way of conversion from Langevin's form to Volterra's form are contained in real part of formula (13). Likewise, we use formulas (5).

Uniqueness

To prove the uniqueness, we address the operator equation (11).

If the coefficients of the equation (11) are measurable functions on a Hilbert space and the fact that:

$$E||n_2(t) - n_1(t) - || \leq c\int_{t_0}^{t}||\xi_2(e) - \xi_1(e)||^2 de$$

where

$$||\xi|| = sup^{Y}||\xi^0||^2,$$

we have:

$$E||\hat{G}(x_2)(t) - \hat{G}(x_1)(t)||^2 \leq c\int_{t_0}^{t}E||x_2(e) - x_1(e)||^2\, de$$

Integrating this assessment, we have:

$$E||\hat{G}^2(x_2)(t)-\hat{G}^2(x_1)(t)||^2 \leq \int_{t_0}^{t} E||\hat{G}(z_2)(e)-\hat{G}(z_2)(e)||^2$$
$$\leq c^2\int_{t_0}^{t} de_1 \int_{t_0}^{t} E||x_2(e)-x_1(e)||^2\, de \leq \frac{c^2(\tau-t_0)^r}{r!}||x_2-x_1||^2$$

And then by induction:

$$||\hat{G}^r(x_2)(t)-\hat{G}^r(x_1)(t)||^2 \leq \frac{c^r(\tau-t_0)}{2!}||x_2-x_1||^2$$

Thus, for sufficiently large r the inequality is:

$$\frac{c^2(\tau-t_0)^r}{r!} < I,$$

then the corresponding map $\hat{G}^r$ is squeezing. This implies the existence of a unique fixed point $\hat{G}$ of the map that is unique (if we identify processes and, that is stochastically equivalent processes).

Conclusion

On the results of this work, we can present the following conclusions:

1. We considered bilinear stochastic singular systems in Langevin's form, which is very ineffective for investigating deterministic nonlinear singular systems.
2. In this work, authors presented the definition and application of apparatus, a new class of pseudo-semi-inverse matrices.
3. As is well known, in the problem statement for solving singular systems, the initial condition stated in our view is noncorrect, or this condition is absent. For the first time, including Sh. S. Smagulov's initial condition for a class of singular nonlinear stochastic systems in bilinear case.
4. This work considered a system as a Map «Input - Output» for a class of systems but with several inputs in Langevin's form.
5. We considered the connection between Langevin's form and Volterra's form.
6. In a bilinear case, we proved the theorem about constructing Volterra`s model for a class of singular nonlinear stochastic systems.
7. Also, the authors proved theorems about uniqueness, convergence, and finiteness (on the class of nilpotent matrices of S. Li) for Volterra's models in Ito`s form in conception of describing a system in form Map «Input - Output» for the aforementioned class of systems, but with several inputs.
8. Due to Sh. S. Smagulov's initial and on-the-base apparatus R.S. Sudakov`s class of pseudo-semi-inverse matrices and use S. L. Sobolev's annulator, we can construct solutions above the name of a class of singular nonlinear stochastic systems in bilinear case.
9. In future, we can solve problems realization, identification, and deterministic and stochastic nonlinear singular systems.

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