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# MODELLING OF TRANSFER MECHANISMS TAKING INTO CONSIDERATION KINEMATIC AND DYNAMIC CHARACTERISTICS

*Abstract*

This paper presents an analysis of mechanical systems. It considers matrix methods for determining the positions of links and the transformation of the simplest movements of the output links of motors into the movements of the working bodies of the machine, which are carried out by a mechanical system consisting of transmission mechanisms. The object of study is a mechanical system including an executive organ, a transmission mechanism and a motor. The technological processes mechanised with the help of this system are used in a wide variety of industries. The practice of engineering calculations demonstrates that in many cases the links of transmission mechanisms are the most pliable, directly transmitting dynamic loads. Consequently, the modelling of transmission mechanisms in mechanical systems, taking into account both kinematic and dynamic characteristics, is a highly relevant area of study. This research presents a set of methods and techniques for determining the kinematic and dynamic characteristics of mechanical systems. It is shown that geometric characteristics significantly affect the dynamics of the mechanical system as a whole. The Lagrangian equation of the second kind is employed in the formulation of the equation of motion of mechanical systems. A crucial phase in the modelling of mechanical systems is the assessment of the stress-strain state of the supporting metallic structure. Autodesk Inventor software was utilised to ascertain the stress-strain state and safety factor of the mechanical system.

**Keywords:** matrix of kinematic pairs, transfer function, transfer mechanism, mechanical system, mathematical model, electromechanical system.

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## КИНЕМАТИКАЛЫҚ ЖӘНЕ ДИНАМИКАЛЫҚ СИПАТТАМАЛАРДЫ ЕСКЕРЕ ОТЫРЫП БЕРІЛІС МЕХАНИЗМДЕРІН МОДЕЛЬДЕУ

*Аңдатпа*

Бұл жұмыста механикалық жүйелерді талдау мәселелері қарастырылған. Буындардың орналасуын анықтаудың матрицалық әдістері, сондай-ақ қозғалтқыштардың Шығыс буындарының қарапайым қозғалыстарын беріліс механизмдерінен тұратын механикалық жүйемен жүзеге асырылатын машинаның жұмыс органдарының қозғалысына айналдыру қарастырылады. Зерттеу объектісі атқарушы органды, беріліс механизмін және қозғалтқышты қамтитын механикалық жүйе болып табылады. Осы жүйенің көмегімен механикаландырылған технологиялық процестер өнеркәсіптің әртүрлі салаларында қолданылады. Инженерлік есептеулер тәжірибесі көрсеткендей, көптеген жағдайларда динамикалық жүктемелерді тікелей жіберетін беріліс механизмдерінің буындары ең икемді болып табылады. Сондықтан кинематикалық және динамикалық сипаттамаларды ескере отырып, механикалық жүйелердің беріліс механизмдерін модельдеу мәселелері өте өзекті. Зерттеу механикалық жүйелердің кинематикалық және динамикалық сипаттамаларын анықтау әдістері мен әдістерінің жиынтығы болып табылады. Геометриялық сипаттамалар жалпы механикалық жүйенің динамикасына айтарлықтай әсер ететіні көрсетілген. Механикалық жүйелердің қозғалыс теңдеуін құруда екінші типтегі Лагранж теңдеуі қолданылады. Механикалық жүйелерді модельдеудің маңызды кезеңі тірек металл құрылымының кернеулі деформацияланған күйін бағалау болып табылады. Механикалық жүйенің кернеулі деформацияланған күйін және қауіпсіздік коэффициентін анықтау үшін Autodesk Inventor бағдарламасы қолданылды.

**Түйін сөздер:** кинематикалық жұптардың матрицасы, беріліс функциясы, беріліс механизмі, механикалық жүйе, математикалық модель, электромеханикалық жүйе.

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# МОДЕЛИРОВАНИЕ ПЕРЕДАТОЧНЫХ МЕХАНИЗМОВ С УЧЕТОМ КИНЕМАТИЧЕСКИХ И ДИНАМИЧЕСКИХ ХАРАКТЕРИСТИК

*Аннотация*

В данной работе рассмотрены вопросы анализа механических систем. Рассматриваются матричные методы определения положений звеньев, а также преобразование простейших движений выходных звеньев двигателей в движения рабочих органов машины, которые осуществляются механической системой, состоящей из передаточных механизмов. Объектом исследования является механическая система, включающая исполнительный орган, передаточный механизм и двигатель. Технологические процессы, механизированные с помощью этой системы, находят применение в самых разных областях промышленности. Практика инженерных расчетов показывает, что во многих случаях наиболее податливыми оказываются звенья передаточных механизмов, непосредственно передающие динамические нагрузки. Поэтому вопросы моделирования передаточных механизмов механических систем, с учетом кинематических и динамических характеристик, являются очень актуальными. Исследование представляет собой совокупность методов и приемов определения кинематических и динамических характеристик механических систем. Показано, что геометрические характеристики существенно влияют на динамику механической системы в целом. При составлении уравнения движения механических систем используется уравнение Лагранжа второго рода. Важным этапом моделирования механических систем является оценка напряженно-деформированного состояния несущей металлической конструкции. Для определения напряженно-деформированного состояния и коэффициента запаса прочности механической системы была применена программа Autodesk Inventor.

**Ключевые слова:** матрица кинематических пар, передаточная функция, передаточный механизм, механическая система, математическая модель, электромеханическая система.

## Main provisions

This paper deals with the analysis of a mechanical system consisting of transmission mechanisms. This mechanical system has both translational and rotational kinematic pairs. Matrix methods are used in determining the position function of the links. The dependence of the output link of the transfer mechanism on the output links of the motors is shown. When considering the mechanism with electric drive, the mechanical system is considered as an electromechanical system. This allows us to derive not only the equation of motion of the mechanical part of the system, but also the associated equations of the electrical part. The study of motion of such systems should be carried out for the whole system or its individual elements. The links of a technological machine can be represented in the form of local models. The paper also considers the case when the link directly transmitting dynamic loads is an elastic element of the transmission mechanism. As a result, we consider a dynamic model with an elastic transfer mechanism. This paper shows that when modelling the transfer mechanisms of mechanical systems, it is necessary to take into account the position functions, elasticity of links, and motor characteristics. To obtain a complete system of equations of motion of the machine, to the equations of motion of the mechanical system must be added to the characteristics of the motor. From the obtained equations, the values of generalised coordinates, generalised velocities and generalised forces are determined.

## Introduction

The problems of the theory of mecha-nisms and machines can be grouped into three areas: analysis of mechanisms, syn-thesis of mechanisms and the theory of automatic machines. Analysis of a mecha-nism consists of studying the kinematic and dynamic properties of a mechanism ac-cording to its given scheme, and synthesis of a mechanism consists of designing a scheme of a mechanism according to its given properties. A lot of research work is being carried out in all three of these areas of the theory of mechanisms and ma-chines.

The article discusses the analysis of the manipulator mechanism of a techno-logical machine Fig. 3.This mechanical system has both translational and rotational kinematic pairs. To determine the position of the links of a mechanical system, you can use graphic, analytical, graph-analytical methods [1-5]. When compiling mathe-matical models of the kinematics of manipulators, matrix calculus has become most widespread. In this work, the matrix method is used to analyze the position of links, and the coordinate transformation equations are shown.

When determining the position function of the links, the theory of matrices of kinematic pairs is considered. The paper considers the dynamic criteria of the kinetostatic model of mechanical systems.The influence of geometric characteristics on the dynamics of a mechanical system is shown. When determining the law of motion of mechanisms, local dynamic models of transmission mechanisms are usually considered, taking into account the characteristics of the engine Fig. 4. The article discusses some issues of compiling mathematical models of electromechanical systems. For the dynamic analysis of the movement of an electromechanical system, as a mechanism with electric drives, the most convenient are the equations Lagrange-Maxwell, which make it possible to obtain equations of motion not only for the mechanical part of the system, but also for the electrical part associated with it. The study of the motion of such systems must be carried out for the whole system or its individual elements [6].

During the study, a scale model of the technological machine manipulator was created Fig. 6. An important stage in the design of mechanical systems is the assessment of the stress-strain state (SSS) of the supporting metal structure. For this purpose, software packages that implement the finite element method are increasingly used [7,8]. In this work, the Autodesk Inventor program was used.

**Research methodology**

The transformation of system coordinates $x_j y_j z_j$ into coordinates $x_i y_i z_i$ is performed according to the equations:

$$\begin{aligned} x_i &= a_{11}x_j + a_{12}y_j + a_{13}z_j + a_i \\ y_i &= a_{21}x_j + a_{22}y_j + a_{23}z_j + b_i \\ z_i &= a_{31}x_j + a_{32}y_j + a_{33}z_j + c_i \end{aligned} \tag{1}$$

where are $a_i, b_i, c_i$ the origins of system j in system i, and the coefficients of the coordinates are the direction cosines of system j relative to i:

$$\begin{aligned} a_{11} &= cos\left(\widehat{x_j, x_i}\right),\ a_{12} = cos\left(\widehat{y_j, x_i}\right),\ a_{13} = cos\left(\widehat{z_j, x_i}\right), \\ a_{21} &= cos\left(\widehat{x_j, y_i}\right),\ a_{22} = cos\left(\widehat{y_j, y_i}\right),\ a_{23} = cos\left(\widehat{z_j, y_i}\right), \\ a_{31} &= cos\left(\widehat{x_j, z_i}\right),\ a_{32} = cos\left(\widehat{y_j, z_i}\right),\ a_{33} = cos\left(\widehat{z_j, z_i}\right), \end{aligned} \tag{2}$$

If during this transformation we keep the designations of the direction cosines according to (2), then the matrix $T_{ij}$ has the form:

$$T_{ij} = \begin{Vmatrix} a_{11} & a_{21} & a_{31} & a_j \\ a_{12} & a_{22} & a_{32} & b_j \\ a_{13} & a_{23} & a_{33} & c_j \\ 0 & 0 & 0 & 1 \end{Vmatrix}$$

If the beginning of system j does not coincide with the beginning of system i, then to measure the Euler angles it is necessary to draw through axes parallel to the axes of system i.$O_j$

Let us introduce the notation:

$cos\theta_{ji} = c_1, cos\psi_{ji} = c_2, cos\phi_{ji} = c_3, sin\theta_{ji} = s_1, sin\psi_{ji} = s_2, sin\phi_{ji} = s_3$ (Fig. 1.)

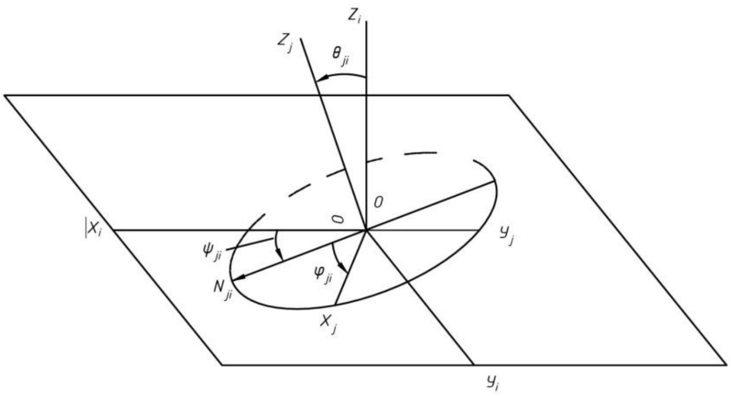


*Fig. 1. Euler angles*

Then the direction cosines:

$$a_{11} = c_2c_3 - c_1s_2s_3,\ a_{12} = -c_2s_3 - c_1s_2c_{3,}\quad a_{13} = s_1s_2, a_{21} = s_2c_3 + c_1c_2s_3,\ a_{22} = -s_2s_3 + c_1c_2c_{3,}\quad a_{23} = -s_1c_2,\quad a_{31} = s_1s_3,\ a_{32} = s_1c_{3,}\quad a_{33} = c_1. \qquad (3)$$

The matrix of coefficients of the right-hand sides of the coordinate transformation equations $T_{ij}$ (or $T_{ji}$) depends only on the type of kinematic pair and therefore can be called the matrix of the kinematic pair. Taking into account the formulas for the connection between direction cosines and Euler angles (3), we obtain:

$$T_{ji} = \begin{Vmatrix} cos\psi_{ji}cos\phi_{ji} - cos\theta_{ji}sin\psi_{ji}sin\phi_{ji} & -cos\psi_{ji}sin\phi_{ji} - cos\theta_{ji}sin\psi_{ji}cos\phi_{ji} & sin\theta_{ji}sin\psi_{ji} & a_i \\ sin\psi_{ji}cos\phi_{ji} + cos\theta_{ji}cos\psi_{ji}sin\phi_{ji} & -sin\psi_{ji}sin\phi_{ji} + cos\theta_{ji}cos\psi_{ji}cos\phi_{ji} & -sin\theta_{ji}cos\psi_{ji} & b_i \\ sin\theta_{ji}sin\phi_{ji} & sin\theta_{ji}cos\phi_{ji} & cos\theta_{ji} & c_i \end{Vmatrix} \qquad (4)$$

For links i and j of a rotational pair, we direct the axis $O_iz_j$ along the axis of this pair, the shortest distance $l_i$ between the axes $O_iz_i$ and $O_jz_j$ is compatible with the axis $O_ix_i$, and the origin of coordinates $O_j$ is placed at a distance $l_{ji}$ from the axis $O_ix_i$ (Fig. 2).

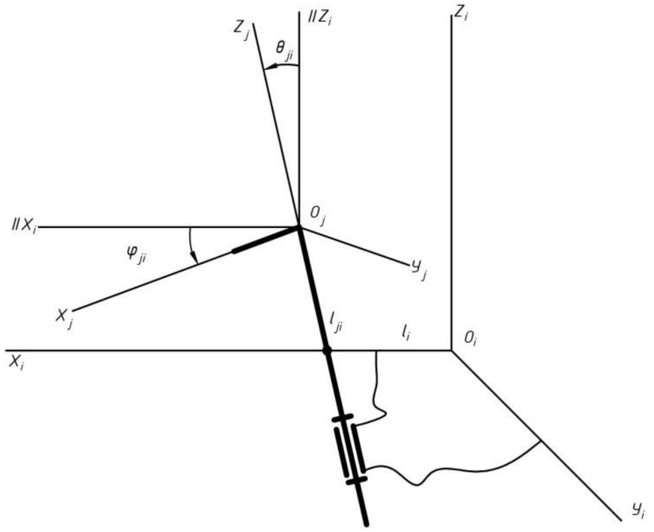


*Fig. 2 Rotational pair*

Then the nutation angle $\theta_{ji} = const$, the procession angle $\psi_{ji} = 0$, and taking into account the accepted notation we obtain from (4) the matrix of the rotational pair:

$$T_{ji} = \begin{Vmatrix} cos\phi_{ji} & -sin\phi_{ji} & 0 & l_i \\ cos\theta_{ji}sin\phi_{ji} & cos\theta_{ji}cos\phi_{ji} & -sin\theta_{ji} & -l_{ji}sin\theta_{ji} \\ sin\theta_{ji}sin\phi_{ji} & sin\theta_{ji}cos\phi_{ji} & cos\theta_{ji} & l_{ji}cos\theta_{ji} \end{Vmatrix} \quad (5)$$

Translational pair matrix is obtained from matrix (5), if we consider the parameter $\mathrm{l_{ji}} = \mathrm{s_{ji}}$ to be a variable value, and the angle $\phi_{ji} = 0$. The angle $\theta_{ji}$ in this case is the angle between the axis $O_iz_i$ and the axis of the translational pair, and the value $l_i$ is equal to the shortest distance between these axes.

Under the same conditions, if instead of a translational pair there is a screw, then the distance $l_{ji} = s_{ji}$ should be considered a variable value related to the angle of rotation $\phi_{ji}$ by the relation

$$s_{ji} = h_{ji}\frac{\phi_{ji}}{2\pi}$$

Where $h_{ji}$ is the pitch of the helix.

As in the case of plane mechanisms, the problem of determining the positions of the links of spatial mechanisms by the method of coordinate transformation can be reduced to the joint solution of the equations obtained by opening one or more kinematic pairs. In contrast to the problem of analytical determination of link positions, which is generally reduced to the solution of a system of nonlinear equations, the problem of determining the velocities and accelerations of any points on the links of the mechanism can always be reduced to the solution of a system of linear equations. Consequently, this problem is not difficult.

**Results of the study**

The transformation of the simplest motions of output links of engines into motions of working bodies of the machine is carried out by a mechanical system consisting of transmission mechanisms. The transformation of motion performed by the transmission mechanisms is characterised by position functions.

At the initial stage of the study, it is necessary to linearise certain nonlinear parameters of the motor and transmission mechanisms, including friction in kinematic pairs, the elastic properties of links, and so forth.

In the case when we consider that the elements are absolutely rigid, there are no gaps and the system has one degree of mobility, the position of any element is uniquely determined from the angle of rotation of the driving link$\varphi_1$.

Then

$$\varphi_n = \Pi_n(\varphi_1)$$

Where $\Pi_n$ is the position function of link n.

Transfer functions or analogues of speeds, accelerations and jerk are

$$\Pi'_n = \frac{d\Pi_n}{d\varphi_1}, \quad \Pi''_n = \frac{d^2\Pi_n}{d\varphi_1^2}, \quad \Pi'''_n = \frac{d^3\Pi_n}{d\varphi_1^3}.$$

The connection $\dot{\varphi}_n, \ddot{\varphi}_n, \dddot{\varphi}_n$ between these geometric characteristics and kinematic ones is determined

$$\begin{cases} \dot{\varphi}_n = \Pi'_n(\varphi_1)\dot{\varphi}_1, \ \ddot{\varphi}_n = \Pi''_n(\varphi_1)\dot{\varphi}_1^2 + \Pi'_n(\varphi_1)\ddot{\varphi}_1, \\ \dddot{\varphi}_n = \Pi'''_n(\varphi_1)\dot{\varphi}_1^3 + 3\Pi''_n(\varphi_1)\dot{\varphi}_1\ddot{\varphi}_1 + \Pi'_n(\varphi_1)\dddot{\varphi}_1. \end{cases} \quad (6)$$

From (6) it is clear that when using transfer functions there is a clear separation of geometric and kinematic characteristics. In gear mechanisms with round wheels, the gear ratio is constant

$$i - const, \qquad \frac{d\varphi_n}{d\varphi_1} = k, \quad \varphi_n = k\varphi_1 + C$$

the position function is linear.

$$\dot{\varphi}_n = \Pi_n'(\varphi_1)\dot{\varphi}_1, \ \ddot{\varphi}_n = \Pi_n'(\varphi_1)\ddot{\varphi}_1, \dddot{\varphi}_n = \Pi_n'(\varphi_1)\dddot{\varphi}_1$$

Inertial loads occur only when the condition or is violated $\varphi_1 - \text{const}$, $\Pi_n' - \text{const}$. With a nonlinear position function (cam, lever, stepping, etc.), even in an ideal mechanism $\ddot{\varphi}_n \neq 0$, inertial loads arise. If, for example, a force $\bar{F}$ is applied on the driven link n, which is balanced on the driving link by the moment M, then, due to the equality of work on possible displacements

$$M = \Pi_n'(\varphi_1)F$$

$$Md\varphi_1 = Fds_n = \Pi_n'(\varphi_1)d\varphi_1 F, \ s_n = \Pi_n(\varphi_1), \ ds_n = \Pi_n'(\varphi_1)d\varphi_1 \qquad (7)$$

It is obvious that even $\Pi_n' \neq \text{const}$ a constant force $\bar{F}$ leads to the appearance of an alternating disturbing moment on the driving link, capable of exciting forced oscillations of the drive. Let us assume that the force $\bar{F}$ is the inertia force of the driven link n, assuming that the driven link performs translational motion at $\dot{\varphi}_1 - \text{const}$, we have

$$\dot{s}_n = \Pi_n'(\varphi_1)\dot{\varphi}_1, \ \ddot{s}_n = \Pi_n''(\varphi_1)\dot{\varphi}_1^2 + \Pi_n'(\varphi_1)\ddot{\varphi}_1 \qquad \text{если } \dot{\varphi}_1 - const$$

$$|F| = |F_n| = m|\ddot{s}| = m|\Pi_n''(\varphi_1)|\dot{\varphi}_1^2 \qquad (8)$$

substituting into (7), we have

$$|M| = m\dot{\varphi}_1^2|\Pi_n'\Pi_n''| \qquad (9)$$

From here

$$T_n = \frac{m\dot{s}^2}{2}, \qquad \frac{dT}{dt} = m\dot{s}\ddot{s}, \quad \frac{dT}{dt} = m\Pi_n'\Pi_n''\dot{\varphi}_1^3.$$

$$\Pi_n'\Pi_n'' = \frac{1}{m\dot{\varphi}_1^3}\frac{dT}{dt}, \qquad \text{where } \frac{dT}{dt} - \text{kinetic power}$$

Expressions (8-9) indicate that geometric characteristics significantly influence the dynamics of the mechanical system. Therefore, extreme values $|\Pi_n'|_{\max}$, $|\Pi_n''|_{\max}$, $|\Pi_n'\Pi_n''|_{\max}$, of functions can be used as dynamic criteria with the help of which various laws of motion are compared, as well as the synthesis of new laws that have, in a certain sense, optimal properties.

In the following, references to position functions and transfer ratios are always provided when constructing dynamic models of mechanisms. Consequently, the laws of motion of the initial links in force analysis are assumed to be specified. In this context, the position function expressions of typical transfer mechanisms and kinematic analysis algorithms are presented in this section.

Now lets return to the consideration of the manipulator of the technological machine Fig. 3.

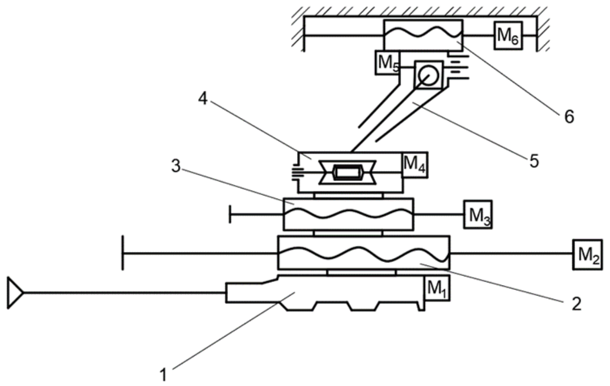


*Fig. 3. Technological machine manipulator*
*1 – working body; 2 – automatic feeder of the working body; 3 - mechanism for moving the automatic feeder forward and backward; 4 - mechanism for rotating the automatic feeder in the horizontal plane; 5 – mechanism for rotating the automatic feeder in the vertical plane; 6- mechanism for moving the manipulator forward and backward; Mi – drive systems.*

The links of the manipulator of the technological machine can be represented as local models, which include the dynamic characteristics of the motor Fig. 4.

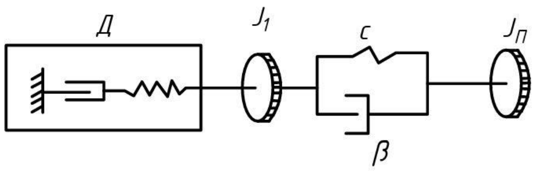


*Fig. 4. The simplest model, including dynamic characteristics of the engine*

In motors, the speed of the output link depends not only on the value of the input parameter u, but also on the load, characterised by the magnitude of the force torque. The static characteristic of the motor can be represented in the form:

$$M_{Д} = M_{\text{СТ}}(u, \dot{q}_0),$$

where $\dot{q}_0 -$ speed of the motor output link.

The practice of engineering calculations shows that in many cases the links of transmission mechanisms directly transmitting dynamic loads are the most ductile. An elastic element of a transmission mechanism (Fig. 3) can be, for example, a travelling screw, which is a rod operating in tension or compression. The stiffness of such an element:

$$c = EF/l,$$

where E is the elastic modulus of the first kind; F is the cross-sectional area; l is the rod length.

Let us consider the reduced moments of inertia of the mechanism (Fig. 4) of the motor $J_1(q_0)$ and actuators в $J_{\Pi}(q_1)$, the reduced moments of driving forces $M_{Д}$ and resistance forces $M_c(q_1, \dot{q}_1)$. Let us denote by c and β the stiffness and the resistance coefficient of the elastic transmission mechanism reduced to its input link. The coefficient of proportionality β is usually determined experimentally. As a result, we come to a dynamic model with an elastic transmission mechanism (Fig. 5). The difference $q_0 - q_1 = \theta$ represents the deformation of the transmission mechanism reduced to the motor output.

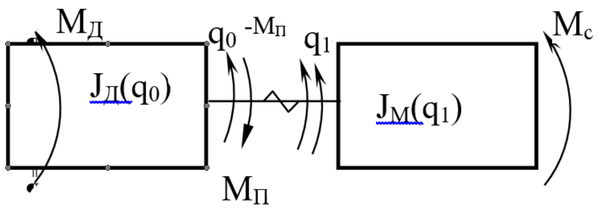


*Fig.5 Dynamic model with elastic transmission mechanism*

Let us make the equations of motion for the system in Fig. 5. For this purpose, the equations of motion of the mechanical systems of the engine and actuators are represented in the form of Lagrange equations of the second kind, taking into account that the moment $M_П$ arising in the transfer mechanism can be regarded as the moment of resistance forces acting on the output link of the mechanical system of the engine, and as the driving moment applied to the input link of the actuators. By analogy with the equation

$$J(q)\ddot{q} + \frac{1}{2}\frac{dJ}{dq}(q)\dot{q}^2 = M_Д + M_c(q, \dot{q})$$

we have

$$J_1(q_0)\ddot{q}_0 + \frac{1}{2}\frac{dJ_1}{dq_0}(q_0)\dot{q}_0^2 = M_Д - M_П = M_Д - \beta(\dot{q}_0 - \dot{q}_1) - c(q_0 - q_1)$$

$$J_M(q_1)\ddot{q}_1 + \frac{1}{2}\frac{dJ_M}{dq_1}(q_1)\dot{q}_1^2 = M_П + M_c(q, \dot{q}) = \beta(\dot{q}_0 - \dot{q}_1) + c(q_0 - q_1) + M_c(q, \dot{q})$$

In order to obtain a complete system of equations of motion of the machine, the motor characteristic must be added to these equations. From the resulting three equations the unknowns $q_0$, $q_1$, $M_Д$.can be determined.

It can be posited that mechanisms with electric drive can be considered as electromechanical systems. For the study of their dynamics, the most convenient methodological approach is the use of Lagrange-Maxwell equations, which have the form of Lagrange equations of the second kind. This approach allows the automatic derivation of not only the equations of motion of the mechanical part of the system, but also the associated equations of the electrical part. The compilation of these equations assumes that the state of an electromechanical system with holonomic connections is determined by the generalised coordinates of the mechanical and electrical parts of the system. The Lantrage-Maxwell equations for holonomic systems have the form [1].

$$\frac{d}{dt}\frac{dL}{\partial \dot{q}_1} - \frac{dL}{\partial q_1} = Q_L \quad l = 1,2 \dots, n$$

$$\frac{d}{dt}\frac{dL}{\partial \dot{\alpha}_k} - \frac{dL}{\partial \alpha_k} = Q_k \quad k = 1, 2, \dots, m$$

Where $L = L_e + L_m$ the Langrange-Maxwell function, $\dot{q}_1$ is the generalized speed, $\dot{\alpha}_k$ is the generalized current, α is the amount of electricity. For the mechanical part $L_m = T - V$, where T is kinetic energy, V is potential energy.

The electrical part of the function for a mechanical system is the same as the magnetic energy

$$L_e = \frac{1}{2}\sum_{r,s} L_{rs} i_r i_s$$

Where r, s are the indices of independent electrical circuits (turns, windings) through which flow, $i_r i_s L_{rs}$ - mutual inductance, with r = s - inductance.

The generalized force $Q_k$ is determined from the variation of the electrical generalized coordinate in the work expression δA electrical forces from the expression

$$\delta A = \sum_{k=1}^{m} \left[ \sum_{r,s} \{ E_{r,s} - R_{r,s,i_{r,s}} \} \right] \delta \alpha_k$$

Where is the $E_{r,s}$ - e.m.f. circuit, $R_{r,s,}$ - circuit resistance.

A crucial phase in the design of mechanical systems is the assessment of the stress-strain state (SSS) of the supporting metal structure. During the course of the study, a scale model of the process machine manipulator (Fig. 6) was constructed. The following data were introduced: Motor holding torque 200.00 Nm; Amperage: 10 A; Full pitch 1.8°; Output shaft diameter 20 mm; Link length: 1400 mm. It is recommended that scale models be employed to investigate the stress-strain state of the system and the kinematics of manipulators.

*Fig. 6. Scale model of the technological machine manipulator*

The utilisation of software packages implementing the finite element method is becoming increasingly prevalent in the evaluation of SSS. The following results were obtained from the research conducted:

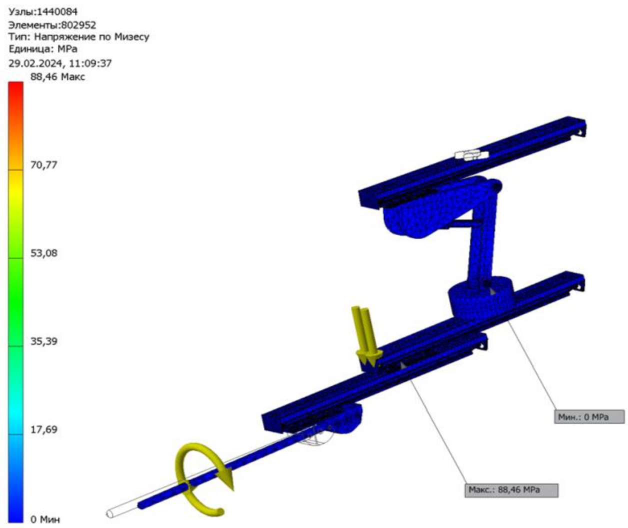


*Fig. **7.** Result of Mises stress*

In Fig. 7 shows that the maximum von Mises stress was 88.46 MPa. The figure shows that there is tension at the joints of the mechanism links, which can have a negative effect at maximum loads.

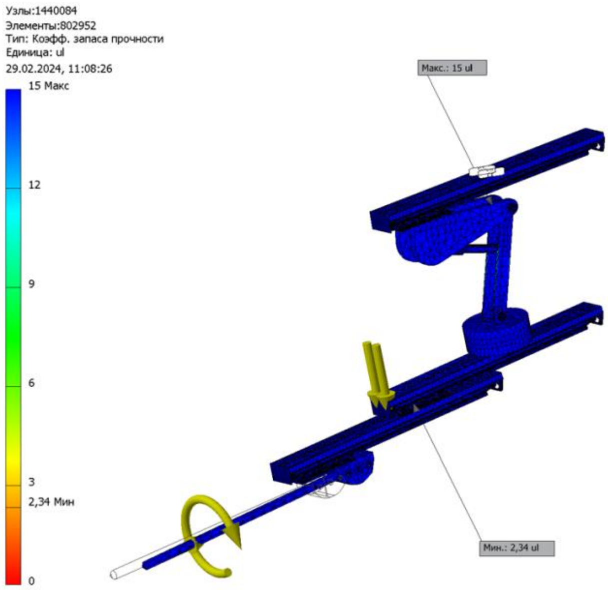


*Fig.8. Safety factor of structures*

After determining the maximum stresses in the structure using the finite element method, we can determine the safety factor of this structure Fig. 8. The minimum safety factor was 2.34. According to GOST requirements, the safety factor must be at least 1.5, so the study showed that this assembly will withstand the load. Therefore, we can use carbon steel material, which will ensure reliable operation of the structure.

**Discussion**

It is typical to assume that the external forces applied to the links of the mechanism are also specified, and thus only the reactions in kinematic pairs are subject to determination. In certain instances, however, external forces applied to the initial links are deemed to be unknown. Subsequently, the force analysis entails the determination of forces at which the accepted laws of motion of the initial links are fulfilled. In both cases, the Dalembert principle is employed, according to which a link of the mechanism can be considered in equilibrium if all external forces acting on it are added to the forces of inertia. When selecting a dynamic model of a functional component of the machine, it is essential to identify those properties that are crucial for the intended purpose and to disregard any characteristics that may be deemed inconsequential. It is therefore to be expected that the same object, the same functional part of the machine, can be described by different dynamic models in different cases.

The most basic dynamic model, which assumes that elements are non-deformable, is typically covered in the introductory course on the mechanics of machines. The results obtained on the basis of this model are conventionally designated as "ideal". In such cases, the problem of determining the inertial forces at a given motion of the links is typically solved.

The second problem of dynamics must be solved only when considering the machine unit as a whole, in connection with the determination of non-uniformity of rotation of the leading links. The analysis of such a kinetostatic model provides insight into the dynamics of mechanisms, which is sufficient for static loading.

The fundamental assumption underlying the dynamic calculations of the classical theory of mechanisms and machine dynamics is the assumption of non-deformability of links. However, the practice of operating machine units indicates that when modelling the working processes of mechanical systems, it is necessary to take into account the variability of parameters, nonlinearity of the position function, elasticity of links, and the characteristics of motors. In the context of modelling technological machines, it is possible to consider the development of mathematical models of the complex system "Executive body - Manipulation system - Transport-technological machine - Support base - Environment" [9].

**Conclusion**

This paper presents an analysis of mechanical systems. This paper considers matrix methods for determining the positions of links and transforming the simplest motions of output links of motors into motions of working bodies of the machine. The research presents a set of methods and techniques for determining the kinematic and dynamic characteristics of mechanical systems.

The fundamental methodologies employed in the investigation of the kinematics and dynamics of robots are elucidated in [10]. Theoretical methods for investigating the dynamics and strength of mechanical systems are based on the construction of mathematical models using the laws of classical mechanics. Newton's mechanics and Lagrange mechanics. The aforementioned issues can be addressed in [11-15], which considers the impact of nonlinearity in system position functions, elasticity of links, variability of parameters within the system, the influence of link and kinematic pair design features, and the dynamics of actuators and actuating mechanisms.

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