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APPLICATION OF BAYESIAN ANALYSIS FOR COMPREHENSIVE ASSESSMENT OF THE STATE OF FISH RESOURCES IN FRESHWATER ECOSYSTEMS

Abstract

Water resources, including rivers, lakes and reservoirs, play a key role in maintaining the ecological balance, developing fisheries and preserving biodiversity. To ensure effective management and rational use of water resources, scientific research is conducted annually, including control fishing, which is aimed at determining the permissible catch volumes and developing strategies for preserving the aquatic environment. Within the framework of such studies, special attention is paid to water quality, the state of the food supply and the analysis of fish populations, including species, numbers and age structure. This study uses a Bayesian approach that reveals complex patterns and dynamic changes in ecosystems that remain invisible when using traditional statistical methods. This approach contributes to a deeper understanding of processes occurring in water bodies and to the optimization of fisheries management. Bayesian analysis allows for more accurate predictions of changes in fish populations and their structures, which contributes to improved management of aquatic ecosystems and the conservation of biodiversity.

Keywords: Bayesian analysis, water resources management, monitoring, biodiversity, fisheries.

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ТҰШЫ СУ ЭКОЖҮЙЕЛЕРІНДЕГІ БАЛЫҚ РЕСУРСТАРЫНЫҢ ЖАҒДАЙЫН КЕШЕНДІ БАҒАЛАУ ҮШІН БАЙЕС ТАЛДАУЫН ҚОЛДАНУ

Аңдатпа

Су ресурстары, соның ішінде өзендер, көлдер мен су қоймалары экологиялық тепе-теңдікті сақтауда, балық шаруашылығын дамытуда және биологиялық әртүрлілікті сақтауда шешуші рөл атқарады. Су ресурстарын тиімді басқаруды және ұтымды пайдалануды қамтамасыз ету үшін жыл сайын балық аулаудың қолайлы көлемін анықтауға және су ортасын сақтау стратегиясын әзірлеуге бағытталған бақылау балық шаруашылығын қоса алғанда, ғылыми зерттеулер жүргізіледі. Мұндай зерттеулерде су сапасына, азық-түлікпен қамтамасыз ету жағдайына және балық популяциясының, оның ішінде түрлерінің, санының және жас құрылымының талдауына ерекше көңіл бөлінеді. Бұл зерттеу дәстүрлі статистикалық әдістерді қолдану арқылы көрінбейтін экожүйелердегі күрделі заңдылықтар мен динамикалық өзгерістерді ашу үшін Байес әдісін пайдаланады. Бұл тәсіл су объектілерінде болып жатқан процестерді тереңірек түсінуге және балық ресурстарын басқаруды оңтайландыруға ықпал етеді. Байес талдауы балықтар популяциясының және олардың заңдылықтарының өзгеруін дәлірек болжауға мүмкіндік береді, бұл су экожүйелерін басқаруды жақсартуға және биологиялық әртүрлілікті сақтауға әкеледі.

Түйін сөздер: Байес талдауы, су ресурстарын басқару, мониторинг, биоәртүрлілік, балық шаруашылығы.

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ПРИМЕНЕНИЕ БАЙЕСОВСКОГО АНАЛИЗА ДЛЯ КОМПЛЕКСНОЙ ОЦЕНКИ СОСТОЯНИЯ РЫБНЫХ РЕСУРСОВ В ПРЕСНОВОДНЫХ ЭКОСИСТЕМАХ

Аннотация

Водные ресурсы, включая реки, озера и водоемы, играют ключевую роль в поддержании экологического баланса, развитии рыболовства и сохранении биоразнообразия. Для обеспечения эффективного управления и рационального использования водных ресурсов ежегодно проводятся научные исследования, включающие контрольные ловы рыбы, которые направлены на определение допустимых объемов вылова и разработку стратегий сохранения водной среды. В рамках таких исследований особое внимание уделяется качеству воды, состоянию кормовой базы и анализу рыбных популяций, включая виды, численность и возрастную структуру. В данном исследовании используется байесовский подход, который позволяет выявить сложные закономерности и динамические изменения в экосистемах, которые остаются незаметными при применении традиционных методов статистики. Этот подход способствует более глубокому пониманию процессов, происходящих в водоемах, и оптимизации управления рыбными ресурсами. Байесовский анализ позволяет более точно прогнозировать изменения в популяциях рыб и их структурах, что способствует улучшению управления водными экосистемами и сохранению биоразнообразия.

Ключевые слова: Байесовский анализ, управление водными ресурсами, мониторинг, биоразнообразие, рыболовство.

Main provisions

The study analyzes fish population dynamics in Kazakhstan using a Bayesian approach, which reveals hidden patterns that are difficult to detect with traditional methods. The results show changes in fish abundance and age, helping to assess population sustainability and improve water resource management. Special attention is given to monitoring the food base and water quality, which influence fish development. Bayesian analysis enhances predictions of changes and optimizes fishery management, contributing to the preservation of ecosystems and biodiversity.

Introduction

The Republic of Kazakhstan possesses significant water resources, including more than 5,000 rivers, lakes, and other bodies of water, which play a crucial role in maintaining ecological balance, developing fisheries, and preserving biodiversity [1]. These aquatic ecosystems are not only vital natural resources but also play a decisive role in the sustainable socio-economic development of the region. They provide a variety of ecosystem services, such as water supply, maintenance of biological diversity, and play a central role in the fishing industry, which is an important sector of the economy [2, 3].

To ensure effective management and rational use of water resources, scientific research is conducted annually, including fish stock assessments, which allow for the evaluation of population status and resilience [4, 5]. These studies aim to establish permissible catch volumes and develop and implement strategies for the conservation of aquatic environments [6]. A crucial part of such research is the monitoring of fish populations, which provides data on species composition, abundance, and age structure, as well as identifying changes in their ecosystems. This data forms the basis for management decisions aimed at the conservation and sustainable use of water resources [7].

One of the important aspects of environmental monitoring is the use of modern statistical methods, including the Bayesian approach. Bayesian analysis is a contemporary method for statistical inference that allows combining prior information about a parameter with new data to obtain an updated understanding of the parameter's value. This method is particularly useful in conditions of uncertainty and complexity, as it allows for the consideration of both prior knowledge and newly acquired data.

The advantages of Bayesian analysis include the integration of prior knowledge, sequential learning, flexibility, and adaptability [8-10].

Bayesian analysis allows for modeling and accounting for uncertainties in data, which enables more accurate predictions of changes in fish populations and the identification of hidden patterns in the data that are not always apparent when using traditional statistical methods [11-13]. This method provides a more comprehensive understanding of potential changes in aquatic ecosystems and can become an important tool in fishery management. The use of the Bayesian approach also allows for the consideration of changes in population structure and more precise forecasting of their future development [11, 14].

The application of Bayesian analysis in assessing the dynamics of fish populations and their ecosystems improves long-term planning in water resource management and provides opportunities for optimizing fisheries and developing measures to conserve the biodiversity of water bodies. It also contributes to enhancing the quality of scientific forecasts and increasing the accuracy of management decisions in fisheries and ecology [9, 13]. Thus, the use of the Bayesian approach enhances the efficiency of monitoring, forecasting, and managing fish resources, promoting the sustainable development of aquatic ecosystems and ensuring the safety of fisheries in Kazakhstan.

Research Methods

This study used a Bayesian approach to analyse data on fisheries catches and age structure of fish populations. Bayesian analysis was chosen as the main research tool due to its ability to account for uncertainty in the data and use prior knowledge, which allows for more accurate modeling of trends and forecasts. The basis of Bayesian statistics is the following formula

$$P(\theta|D) = \frac{P(\theta|D)P(\theta)}{P(D)},$$

where $P(\theta|D)$ represents the posterior distribution of parameters θ for given data D , $P(\theta|D)$ describes the likelihood of the data, $P(\theta)$ is the prior distribution of the parameters, and $P(D)$ acts as a normalization constant that ensures the correctness of the probability distribution.

To model the age structure of fish populations, a normal distribution was used, which allows for flexible description of variable indicators such as mean age:

$$Age \sim \mathcal{N}(\mu, \sigma^2)$$

where μ is the mean age value and σ^2 is the variance characterizing the degree of variability.

The analysis of data on the proportions of different species in the test catches was carried out using a categorical distribution that can adequately account for the probabilities of proportional representation of species:

$$Share\ of\ fish\ species \sim Categorical(p_1, p_2, \dots, p_k)$$

where $p = [p_1, p_2, \dots, p_k]$ are probabilities corresponding to k different fish species.

The Markov Chain Monte Carlo (MCMC) method was used to estimate the parameters, which allows iterative generation of samples from complex posterior distributions. The expected value of the parameter θ was approximated based on the sample mean:

$$\mathbb{E}[\theta] \approx \frac{1}{N} \sum_{i=1}^N \theta_i,$$

where N is the total number of samples and θ_i are the individual parameter values obtained from the posterior distribution.

The main advantage of the Bayesian method lies in its ability to handle uncertainty, which is crucial when analyzing ecological data, where there can be various sources of errors and variability. Unlike classical methods such as linear regression, the Bayesian approach allows for the consideration of probabilistic distributions for all model parameters, providing a more comprehensive understanding of potential trends and changes in fish populations.

To analyze the collected data, a Python script was developed, which automated data processing and model building. The primary library used for Bayesian analysis was PyMC, which allows for the creation of complex statistical models and sampling from the posterior distribution of parameters using the Markov Chain Monte Carlo (MCMC) method [10].

The MCMC method allows for obtaining samples of model parameters and estimating their distributions. This is particularly important when data is limited and it is necessary to account for the variability of parameters in the model.

The main focus was on identifying trends based on two key indicators:

1. Average age of fish – an indicator reflecting the age structure of populations, allowing for the assessment of reproduction dynamics and generational turnover.
2. Proportion of a specific species in control catches – a relative indicator characterizing the distribution of populations and their dominant position in the ecosystem.

It should be noted that the indicators of average age and quantity in the control catch are independent and assess the state of fish resources indirectly. The coincidence of trends in changes in average age and the share of species in catches serves as an additional indicator of the state of fish resources. For example, an increase in average age combined with a decrease in the share of a specific fish species in the control catch may indicate a slowdown in population reproduction, while their opposite dynamics may indicate an improvement in environmental conditions or the effectiveness of regulatory measures.

The study is based on scientific fishing data conducted on the Yesil River from 2014 to 2023. The analysis included an assessment of the average age of fish in catches and the proportional distribution of the main species in control samples. The tabular materials contain information on such species as Roach, Bream, Pike, Common bass, and Tench (Tables 1-2). The presented data were used to perform analytical calculations and prepare reporting materials intended for submission to the LLP «Fisheries Research and Production Center».

Table 1. Average age of fish in the control catch, years

<i>Year</i>	<i>Roach</i>	<i>Bream</i>	<i>Pike</i>	<i>Common bass</i>	<i>Tench</i>
2014	3.98	4.67	3.41	3.80	5.92
2015	3.57	4.11	3.66	3.64	5.00
2016	2.93	3.73	2.87	3.09	6.67
2017	2.86	3.48	2.65	3.06	4.70
2018	2.94	2.53	2.40	2.76	3.37
2019	3.17	2.78	2.56	3.17	3.92
2020	3.75	3.67	3.17	3.43	4.07
2021	3.11	2.99	2.89	2.98	3.37
2022	4.13	3.91	2.88	4.36	5.30
2023	3.48	4.27	3.69	4.08	6.67

Table 2. Share of fish in the control catch, %

Year	Roach	Bream	Pike	Common bass	Tench	Total
2014	36.01	13.67	6.94	35.14	8.24	100.00
2015	41.80	15.17	5.88	32.51	4.64	100.00
2016	40.10	15.40	3.89	38.07	2.54	100.00
2017	35.92	24.35	3.45	34.54	1.73	100.00
2018	30.29	25.63	6.27	32.97	4.84	100.00
2019	37.48	21.26	6.29	31.70	3.27	100.00
2020	31.94	20.00	5.75	26.64	15.67	100.00
2021	28.38	19.03	7.03	29.26	16.30	100.00
2022	30.71	27.24	8.78	28.34	4.94	100.00
2023	45.39	21.05	4.28	28.29	0.99	100.00

Results

The results of the analysis shown in Figures 1-10, conducted using a Bayesian approach, demonstrate changes in the average age of fish and the share of each species in the control catches from 2014 to 2023. These data allow for the identification of key trends in fish populations and the assessment of the impact of environmental factors on their distribution and age structure.

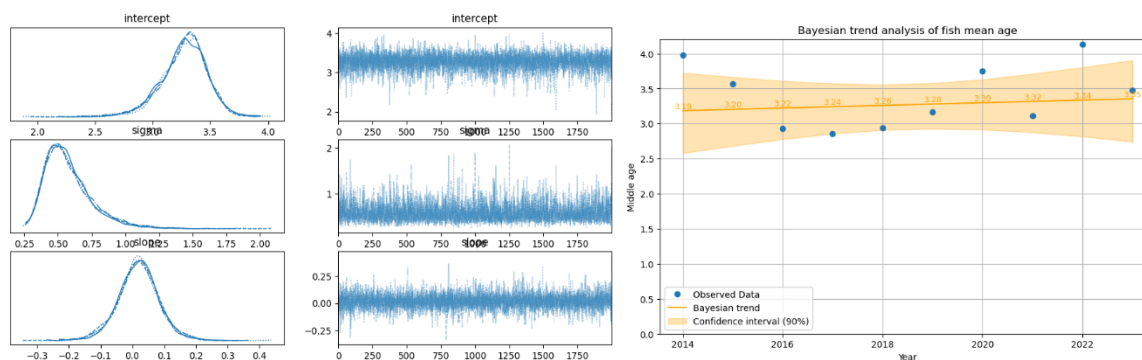


Figure 1. Roach: Trend of average age, years

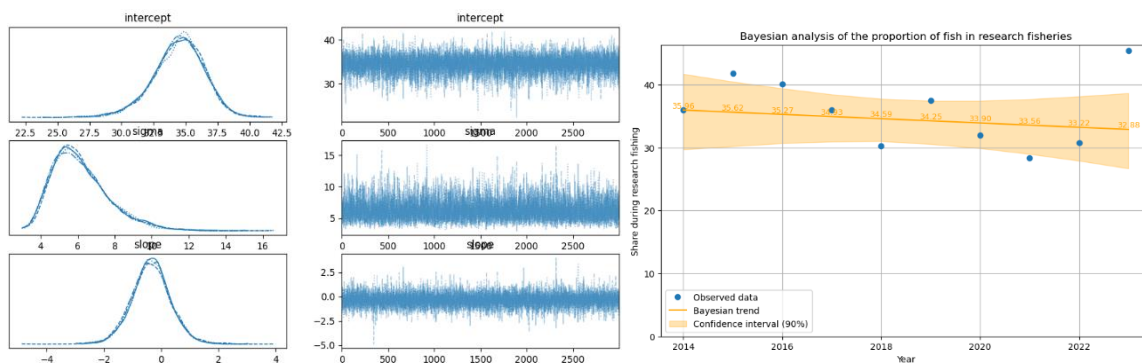


Figure 2. Roach: Trend of share in the control catch, %

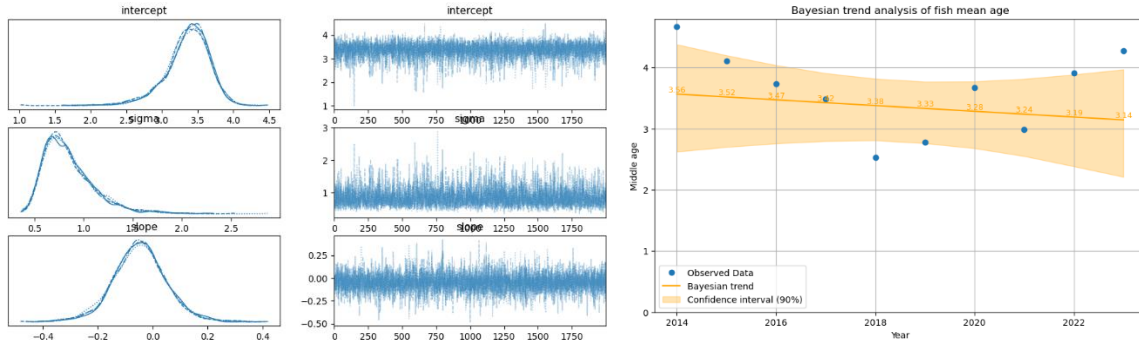


Figure 3. Breem: Trend of average age, years

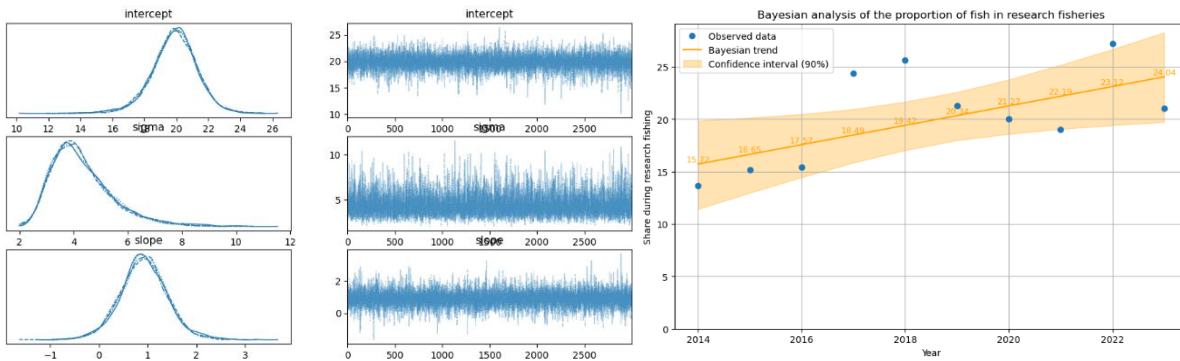


Figure 4. Breem: Trend of share in the control catch, %

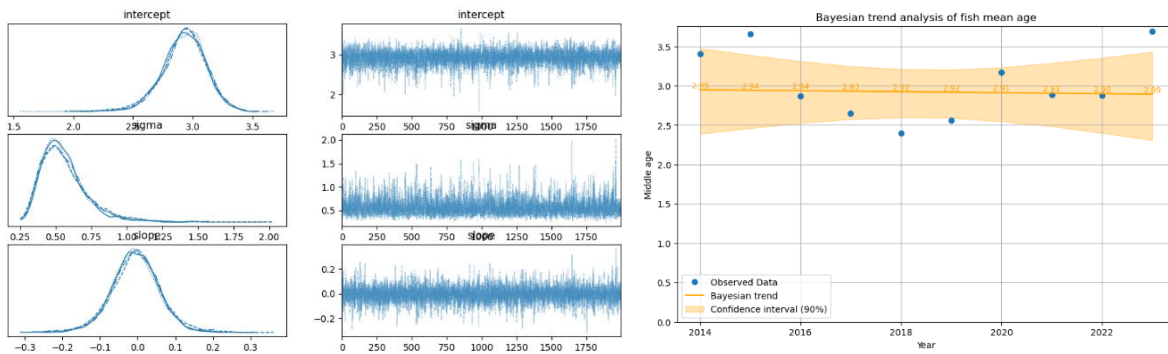


Figure 5. Pike: Trend of average age, years

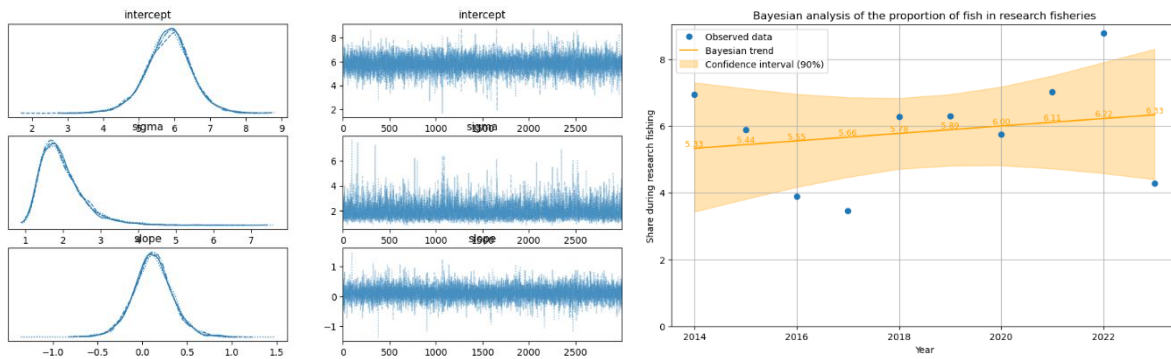


Figure 6. Pike: Trend of share in the control catch, %

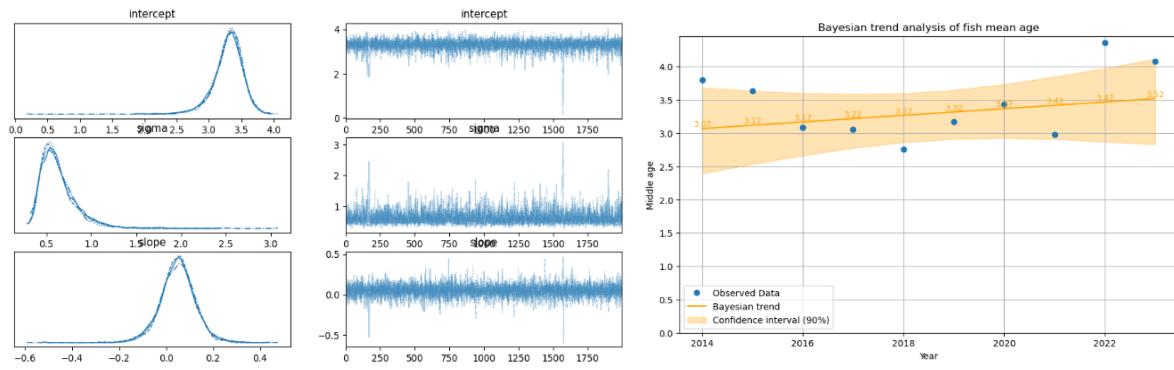


Figure 7. Common bass: Trend of average age, years

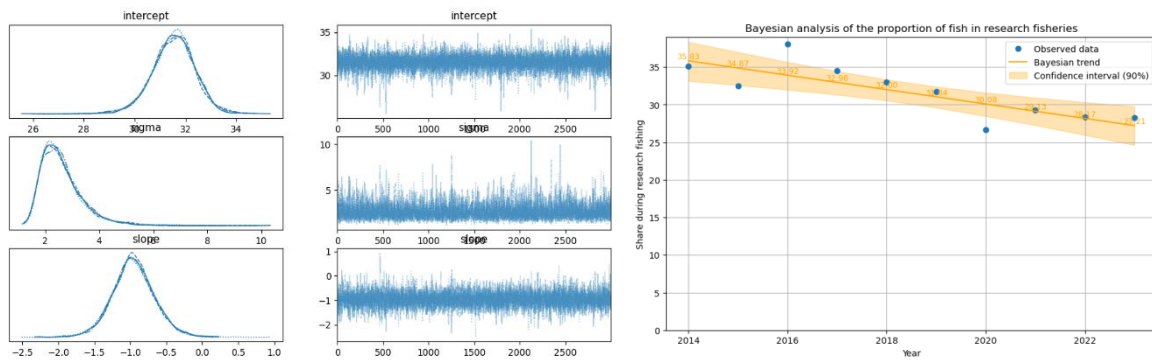


Figure 8. Common bass: Trend of share in the control catch, %

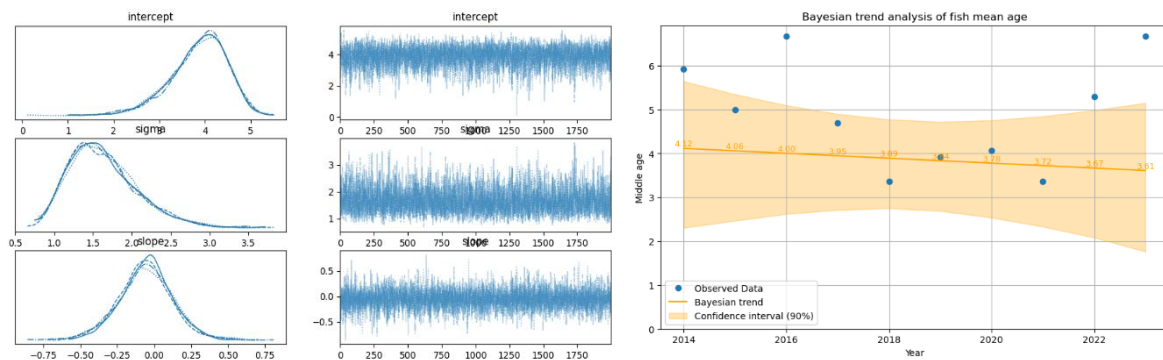


Figure 9. Tench: Trend of average age, years

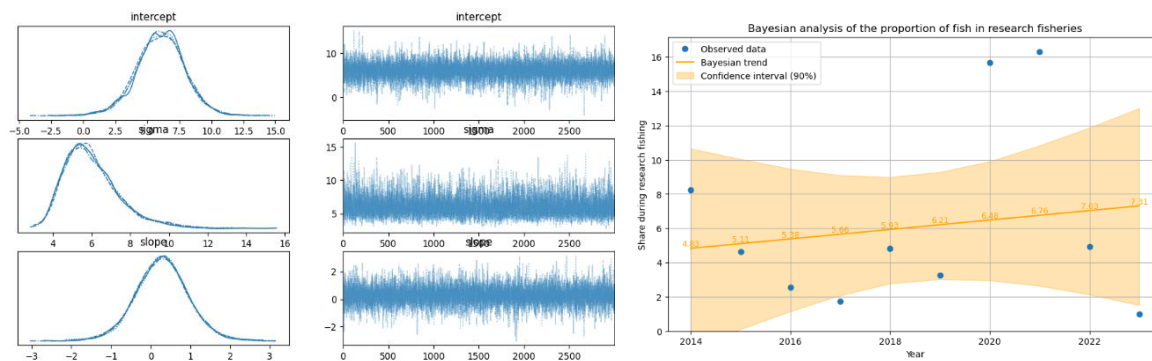


Figure 10. Tench: Trend of share in the control catch, %

The study produced regression lines that reflect the main trend in changes in the average age of fish and the share of each fish species in the control catches. These indicators are independent, and their mutual coincidence allows us to judge the consistency and accuracy of the control methods used to monitor the state of fish populations. Analyzing trends for each of these indicators allows for a deeper understanding of the dynamics of fish resources and the assessment of the effectiveness of catch management.

Additionally, it is important to consider changes in the average age of fish, which should be within the desired range to maintain ecosystem stability. This indicator also requires constant monitoring to promptly identify possible deviations and adjust fishery management measures.

Table 3. Changes in regression line values by year

Fish type	2014		2023		Difference	
	Age	Share %	Age	Share %	Age	Share %
Roach	3.19	36.01	3.55	32.88	0.36	-3.13
Bream	3.56	15.75	3.14	24.00	-0.42	8.25
Pike	2.95	5.31	2.89	6.35	-0.06	1.04
Common bass	3.07	35.82	3.52	27.21	0.45	-8.61
Tench	4.12	4.77	3.61	7.33	-0.51	2.56

In Table 3, a decrease in the share of roach and perch in the control catches can be observed, while their age has also increased. This may indicate a decline in the population of these species in the water body. At the same time, the indicators for bream, pike, and tench show a more positive trend: the share of these species in the catch has increased, and their age in the control catches has become younger. These changes may indicate improved conditions for these species in the water body and more effective reproduction.

Discussion

The results of this study highlight the complexity of fisheries management, even with long-term monitoring data. One of the key problems is the poor quality and incompleteness of the information collected. Data on the abundance and age structure of fish populations often contain significant errors and omissions, which significantly limits the ability to conduct accurate analysis. This, in turn, makes it difficult to develop scientifically sound recommendations for catch management.

In addition, traditional methods of analysis are not effective enough to account for the complex relationships between environmental factors and fish population characteristics. High uncertainty and variability in natural conditions often mean that many key patterns remain hidden. For example, existing data rarely allow direct assessment of the total volume of fish in a water body, which requires more flexible approaches.

In this regard, the use of Bayesian analysis seems to be a promising method. This approach allows for both uncertainties and complex relationships between variables, making it particularly useful for assessing the state of fishery resources in conditions of incomplete data. The use of this method helps to obtain more accurate and reliable results, which can significantly improve the efficiency of fishery resource management.

Conclusion

The study analyzed the dynamics of fish populations in the Yesil River based on long-term data on test catches and the age structure of fish. The Bayesian approach used allowed us to identify key patterns in changes in the state of fish resources, as well as to assess the impact of various factors on their numbers and age.

The results of the analysis showed that for some fish species, such as roach and perch, there is an increase in their age and a decrease in their share in catches, which may indicate a decrease in the number of these species in the reservoir. At the same time, the improvement in indicators for Bream, Pike and Tench, both in terms of share in catches and in age, indicates more favorable conditions for these species. These data are important for developing recommendations for sustainable fisheries and preserving water body ecosystems.

As part of the overall fisheries research in the Republic of Kazakhstan, it is necessary to determine the optimal average age for each fish species depending on the conditions of the water body.

Thus, Bayesian analysis has proven to be an effective tool for assessing the state of fish resources and monitoring their changes, as well as for identifying hidden patterns that would be difficult to notice using traditional methods. The application of such methods in the future will allow for more precise regulation of fish catches, which will contribute to maintaining the ecological balance in the water bodies of Kazakhstan.

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