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R.U. Tuleuova

Kh. Dosmukhamedov Atyrau University, Atyrau, Kazakhstan
*e-mail: tuleuova_79@mail.ru

NUMERICAL STUDY OF THE THERMOELASTIC STATE OF THE ROD IN THE PRESENCE OF A HEAT FLUX SPECIFIED BY A QUADRATIC LAW

Abstract

This paper considers a computational algorithm for the thermoelastic state of a rod clamped at two ends in the presence of a heat flow on its side surface, varying along the coordinates by a quadratic law. The corresponding computational algorithms developed by the author make it possible to solve a class of engineering problems in determining the elongation of rods of limited length. The corresponding computational algorithms developed by the author make it possible to solve a class of engineering problems in determining the elongation of rods of limited length. Therefore, undoubtedly, the results of this work can be widely used in enterprises where calculations are carried out to determine the distribution laws of temperature fields and elongations of structural rod elements operating in a thermal field and in the presence of mechanical forces. The study of compression of a rod of limited length clamped at two ends under the influence of a temperature field and heat flow that varies along the length is of relevant interest for engineering and technological processes related to the strength of partially thermally insulated structural elements operating under coordinate-variable steady-state thermal fields. The relevance of these problems is also motivated by the fact that many structural elements operate under the simultaneous influence of axial force, coordinate-variable heat flow, heat exchange and thermal insulation. With the simultaneous consideration of the above factors, the analytical solution of steady-state thermoelastic compression and tension of rods becomes very complex. In this regard, there naturally arises the need to develop appropriate mathematical models of universal computational algorithms that make it possible to study the thermoelastic state of partially thermally insulated one-dimensional structural elements in the presence of heat exchange with the environment, a variable coordinate of the heat flow field and axial forces.

Keywords: Thermal expansion, Modulus of elasticity, Thermal stress state, Displacement discretisation, Stress strain.

Р.У. Тулеуова

Х. Досмухамедов атындағы Атырау университеті, Атырау қ., Қазақстан

КВАДРАТТЫҚ ЗАҢ МЕНЕН АНЫҚТАЛҒАН ЖЫЛУ АҒЫСЫ АТЫНДАҒЫ ТҰРЫҚТЫҢ ТЕРМОЭЛЕСТИЯЛЫҚ КҮЙІН САНДЫҚ ЗЕРТТЕУ

Аңдатпа

Бұл жұмыста координаталар бойымен квадраттық заң бойынша өзгеретін, оның бүйір бетінде жылу ағыны болған кезде екі ұшынан қысылған өзекшенің термосерпімді күйінің есептеу алгоритмі қарастырылады. Автор әзірлеген сәйкес есептеу алгоритмдері шектелген ұзындықтағы өзекшелердің ұзаруын анықтауда инженерлік есептердің бір класын шешуге мүмкіндік береді. Сондықтан, сөзсіз, бұл жұмыстың нәтижелері жылу өрісінде және механикалық күштер болған кезде жұмыс істейтін құрылымдық өзек элементтерінің температуралық өрістердің таралу заңдылықтарын және ұзартуларын анықтау үшін есептеулер жүргізілетін кәсіпорындарда кеңінен қолданылуы мүмкін. Температуралық өрістің және ұзындығы бойынша өзгеретін жылу ағынының әсерінен екі шетінен қысылған ұзындығы шектелген штанганың қысылуын зерттеу, жартылай жылу оқшауланған құрылымдық элементтердің беріктігіне байланысты инженерлік және технологиялық процестер үшін өзекті қызығушылық тудырады. координаталық айнымалы тұрақты күйдегі жылу өрістері. Табиғи ресурстарды өндіру мен терең өңдеудің қарқынды дамуы ғалымдардың алдында сәйкес математикалық модельдер мен әмбебап есептеу алгоритмдерін жасаудың жаңа мәселелерімен бетпе-бет келді және олардың бүйір беттері бойымен координаталық-айнымалы жылу ағындарының әсерінен ішінара жылу оқшауланған құрылымдық элементтердің термосерпімді күйін кешенді зерттеуге мүмкіндік беретін әдістер - мұның бәрі өте өзекті. Бұл мәселелердің өзектілігі көптеген құрылымдық элементтердің осьтік күштің, координаталық айнымалы жылу ағынының, жылу алмасудың және жылу оқшаулауының бір мезгілде әсер етуімен жұмыс істейтіндігімен де негізделген. Жоғарыда аталған факторларды бір мезгілде қарастырған кезде стержендердің стационарлық күйдегі термосерпімді сығу мен керілуінің аналитикалық шешімі өте күрделі болады. Осыған байланысты, қоршаған ортамен жылу алмасуы, айнымалы координатасы болған кезде жартылай жылу оқшауланған бір өлшемді құрылымдық элементтердің термосерпімді күйін зерттеуге мүмкіндік беретін әмбебап есептеу алгоритмдерінің сәйкес математикалық үлгілерін жасау қажеттілігі табиғи түрде туындайды. жылу ағынының өрісі және осьтік күштер.

Түйін сөздер: термиялық кеңею, серпімділік модулі, жылулық кернеу күйі, ығысу дискретизациясы, кернеу деформациясы.

Р.У. Тулеуова
Атырауский университет имени Х.Досмухамедова, г. Атырау, Казахстан

ЧИСЛЕННОЕ ИЗУЧЕНИЕ ТЕРМОУПРУГОГО СОСТОЯНИЯ СТЕРЖНЯ ПРИ НАЛИЧИИ ТЕПЛОВОГО ПОТОКА, ЗАДАННОГО КВАДРАТИЧНЫМ ЗАКОНОМ

Аннотация

В данной работе рассматривается вычислительный алгоритм термоупругого состояния стержня защемленного двумя концами при наличии теплового потока на ее боковой поверхности, меняющегося по координате квадратичным законом. Соответствующие вычислительные алгоритмы, разработанные автором, позволяют решить класс инженерных задач по определению удлинения стержней ограниченной длины. Поэтому, несомненно, результаты данной работы могут широко применяться на предприятиях, где проводятся расчеты по определению законов распределения температурных полей и удлинений стержневых элементов конструкций, работающих в тепловом поле и при наличии механических сил. Исследование сжатия защемленного двумя концами стержня ограниченной длины, при воздействии переменного по длине температурного поля и теплового потока, представляет соответствующий интерес для техники, технологических процессов связанных с прочностью частично теплоизолированных элементов конструкций, работающих при переменных по координате установившегося теплового поля. Бурное развитие добычи и глубокой переработки природных богатств, ставшие перед учеными новые проблемы разработки соответствующих математических моделей, универсальных вычислительных алгоритмов и методов, позволяющих всестороннее исследование термоупругого состояния частично-теплоизолированных элементов конструкций при воздействии переменной по координате тепловых потоков по их боковым поверхностям все это весьма актуально. Актуальность этих проблем также мотивируется тем, что многие элементы конструкций работают при одновременном воздействии осевой силы, переменного по координате теплового потока, теплообмена и теплоизоляции. При одновременном учете вышеприведенных факторов аналитическое решение установившегося термоупругого сжатия и растяжения стержней становится весьма сложной. В связи с этим, естественным образом появляется необходимость разработки соответствующих математических моделей универсальных вычислительных алгоритмов, позволяющих исследовать термоупругое состояние частично теплоизолированных одномерных элементов конструкций, при наличии теплового обмена с окружающими средами, переменной по координате поля теплового потока и осевых сил.

Ключевые слова: тепловое расширение, модуль упругости, терм-напряженное состояние, дискретность смещения, напряженная деформация.

Main provisions

Modern internal combustion engines, gas turbine power plants, oil heating compressor stations, steam generators of nuclear reactors, and technological processes that allow the deep processing of uranium and osmium ores, as well as crude oil, pose the urgent problem of developing a mathematical model for studying the temperature distribution field of thermal, physico-mechanical state of the bearing elements of these structures, taking into account the nonlinear physical properties of materials and their operating conditions.

It is known that in a thermo-physical-mechanical process, the main characteristic that has a significant effect on the strength of load-bearing structural elements is an intense temperature rise, i.e., heat flux. In general, temperature is one of the most important characteristics of the growth process and affects the morphology and crystal structure of heat-resistant alloys. Temperature fields differ sharply in different parts of the rod uneven. Consequently, during the thermomechanical process, in some areas of the structural elements, the temperature will be acceptable, and in some areas – critical, which leads to rapid wear of the structural elements and to the loss of their physical qualities. In this regard, for an accurate calculation of the distribution of the temperature field over the volume of multidimensional bodies of various configurations made of heat resistant alloys, it is necessary to carry out effective theoretical and numerical modelling [1-4].

The purpose of modelling, both analytical and imitation, is to predict the state of the system, which most realistically displays the picture of the temperature field distribution over the volume of a multidimensional body [5-8]. In the long term, based on this forecast, by changing both the internal parameters of the structure of structural elements and the characteristics of external influences, it will be possible to determine all the vulnerabilities in the structural elements and protect them from deformation or destruction. The development of a model of the temperature distribution over the body volume is necessary, since the complexity of the thermomechanical process in real time greatly reduces the ability to intuitively assess the identification of critical temperatures in body parts [9-12].

Introduction

The article discusses the issues of creating a mathematical model and developing corresponding computational algorithms that allow one to study the thermally stressed state of a partially thermally insulated rod of limited length and rigidly clamped at two ends, when applying heat flows to the side surface, varying in coordinates in the presence of a heat transfer process through the cross-sectional areas of the two clamped ends, based on the energy principle, in combination with the finite element method using quadratic elements with three nodes. We consider a problem where a heat flux is supplied to the side surface of a rod, of limited length and clamped at two ends, varying along the coordinate by a parabolic law. With the above accepted initial data, taking $q(x)=\frac{160}{\ell^2}x^2-\frac{160}{\ell}x$, by the method of solving a static indeterminate problem, we find that $R=-35587{,}0016(кГ)$, $\sigma=-2833{,}36(кГ/см^2)$. A table 1 is given where the convergence of the obtained numerical results to the exact one is ensured by discretizing the rod under consideration by 10 quadratic finite elements. Also for this case the relationship between R, σ and L, h, T_{OC}, and is given in the tables.

Materials and methods

The rod of the limited length L, (cm) is given, both ends of which are rigidly clamped, the cross-sectional area of the rod F, (cm2) is constant along the length of the rod. The physical, mechanical and thermal properties of the rod material are characterized by the modulus of elasticity E, (kG/cm2), the coefficient of thermal expansion, thermal conductivity Kxx, (W/ (cm . °C)) and the coefficient of heat exchange with the environment h, (W/(cm2 . °C)). A heat flux is supplied to the entire length of the lateral surface of the rod, changing along the length of the rod in the following quadratic law

$$q(x)=ax^2+bx+c,\quad a,b,c=const, \tag{1}$$

where a, b, c are real numbers. We will accept the values of the characterizing geometric, physical-mechanical and thermal properties of the rod under consideration as in the previous problem (Figure 1).

When a given heat flux acts on the lateral surface of the rod *q/x, (Вт/см^2)* the rod heats up. Therefore, it must expand. Since both ends of the rod are rigidly clamped, it cannot extend. So, a compressing force *R, (kG)* arises at both ends of the rod, which leads to the appearance of stress in the sections of the considered rod.

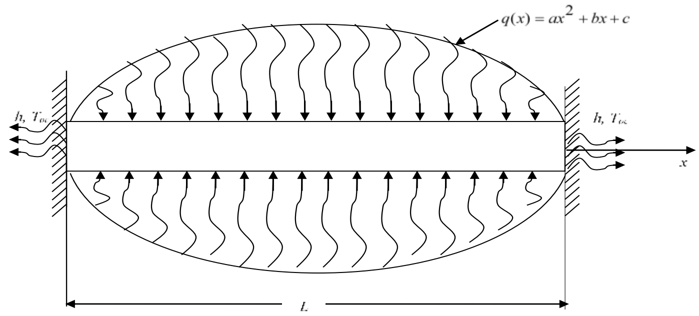


Figure 1. Calculation scheme.

Such a problem is called statically indeterminate. Despite this, this problem can be solved numerically if we use the method of minimizing potential energy in combination with a quadratic finite element with three nodes. By analogy with [3], the expression for the potential energy for the problem under consideration is determined by (7), where *V, (cm^3)* is the volume of a considered rod, $\varepsilon_x = \frac{\partial u}{\partial x}$ is the elastic longitudinal deformation, *u (x), (cm)* is the displacement of the rod points, $\sigma_x = E\varepsilon_x = E\frac{\partial u}{\partial x}, \left(\kappa\Gamma/\text{см}^2\right)$ is the elastic component of the compressive-tensile stress, $T(x)$ is the law of the distribution of the temperature field along the length of the rod.

Let us assume that the field of displacement distributions can be approximated by a second-order curve passing through three points x_i, x_j and x_k. Thus $x_i = 0, x_j = \frac{L}{2}$, $x_k = L$ then a studied rod, taking as one quadratic finite element with three nodes, we have that within the length of the rod

$$u(x) = \frac{L^2 - 3Lx + 2x^2}{L^2}u_i + \frac{4Lx - 4x^2}{L^2}u_j + \frac{2x^2 - Lx}{L^2}u_k . \quad (2)$$

Then the displacement gradient, i.e. the distribution field of elastic deformations, has the following form

$$\varepsilon_x = \frac{\partial u}{\partial x} = \frac{4x - 3L}{L^2}u_i + \frac{4L - 8x}{L^2}u_j + \frac{4x - L}{L^2}u_k . \quad (3)$$

Based on Hooke's law, the value of the elastic component of stress is determined as follows

$$\sigma_x = E\varepsilon_x = E\left(\frac{4x - 3L}{L^2}u_i + \frac{4L - 8x}{L^2}u_j + \frac{4x - L}{L^2}u_k\right), \quad (4)$$

Using the last two expressions, we find the following product

$$\sigma_x\varepsilon_x = E\left(\frac{4x - 3L}{L^2}u_i + \frac{4L - 8x}{L^2}u_j + \frac{4x - L}{L^2}u_k\right)^2 = \frac{E}{L^4}\left[\left(16x^2 - 24Lx + 9L^2\right)u_i^2 + 2\left(40Lx - 32x^2 - 12L^2\right)\times \right.$$
$$\times u_i u_j + 2\left(16x^2 - 16Lx + 3L^2\right)u_i u_k + \left(16L^2 - 64Lx + 64x^2\right)u_j^2 + 2\left(24Lx - 4L^2 - 32x^2\right)u_j u_k + \left(16x^2 - 8Lx + \right.$$
$$\left. + L^2\right)u_k^2\Big]. \quad (5)$$

Assuming that the temperature distribution field along the length of the rod is also approximated by a second-order curve and using the properties of a quadratic finite element with three nodes, we find the expression for the product

$$T(x)\varepsilon_x = \frac{1}{\ell^4}\left[\left(\ell^2 - 3\ell x + 2x^2\right)T_i + \left(4\ell x - 4x^2\right)T_j + \left(2x^2 - \ell x\right)T_k\right]\cdot\left[\left(4x - 3\ell\right)u_i + \left(4L - 8x\right)u_j + \left(4x - L\right)u_k\right]. \quad (6)$$

Now we calculate the first integral in the expression for potential energy (7)

$$\int_V \frac{\sigma_x \varepsilon_x}{2} dV = \frac{E}{2}\int_V \varepsilon_x^2 dV = \frac{EF}{2}\int_0^L \varepsilon_x^2 dx = \frac{EF}{2L^4}\left\{\left[\frac{16x^3}{3} - 12Lx^2 + 9L^2 x\right]u_i^2 + 2\left[20Lx^2 - \frac{32x^3}{3} - 12L^2 x\right]u_i u_j + \right.$$
$$+2\left[\frac{16x^3}{3} - 8Lx^2 + 3L^2 x\right]u_i u_k + \left[16L^2 x - 32Lx^2 - \frac{64x^3}{3}\right]u_j^2 + 2\left[12Lx^2 - \frac{32x^3}{3} - 4L^2 x\right]u_j u_k + \left[\frac{16x^3}{3} - \right.$$
$$\left.\left. - 4Lx^2 + L^2 x\right]u_k^2\right\}\Bigg|_0^L = \frac{EF}{6L}\left[7u_i^2 - 16u_i u_j + 2u_i u_k + 16u_j^2 - 16u_j u_k + 7u_k^2\right]. \quad (7)$$

According to the theory of thermoelasticity, the values of deformation from the temperature field are determined by the following formula

$$\varepsilon_T = -\alpha T(x). \quad (8)$$

The value of the temperature component of voltage is determined by the formula

$$\sigma_T = E\varepsilon_T, \quad (9)$$

thermoelastic stress

$$\sigma = \sigma_x + \sigma_T. \quad (10)$$

It should be noted here that the sum of the coefficients before the nodal displacements is equal to zero. Using (6), we calculate the second integral in the expression for potential energy

$$\int_V \alpha E T(x)\varepsilon_x dV = \alpha EF\int_0^L T(x)\varepsilon_x dx = \frac{\alpha EF}{L^4}\left\{\left[\frac{13L^2x^2}{2} - 6Lx^3 - 3L^3 x + 2x^4\right]T_i u_i + \left[\frac{32Lx^3}{3} - 10L^2x^2 + 4L^3 \times\right.\right.$$
$$\left.\times x - 4x^4\right]T_i u_j + \left[\frac{7L^2x^2}{2} - \frac{14Lx^3}{3} - L^3 x + 2x^4\right]T_i u_k + \left[\frac{28Lx^3}{3} - 6L^2x^2 + 4x^4\right]T_j u_i + \left[8L^2x^2 - 16Lx^3 + 8\times\right.$$
$$\left.\times x^4\right]T_j u_j + \left[\frac{20Lx^3}{3} - 2L^2x^2 - 4x^4\right]T_j u_k + \left[\frac{32L^2x^2}{2} - \frac{10Lx^3}{3} + 2x^4\right]T_k u_i + \left[\frac{16Lx^3}{3} - 2L^2x^2 - 4x^4\right]T_k u_j +$$
$$\left.+\left[\frac{L^2x^2}{2} - 2Lx^3 + 2x^4\right]T_k u_k\right\}\Bigg|_0^L = \frac{\alpha EF}{6}\left[-3T_i u_i + 4T_i u_j - T_i u_k - 4T_j u_i + 4T_j u_k + T_k u_i - 4T_k u_j + 3T_k u_k\right] (11)$$

In the last square bracket, the sum of the coefficients before $T_j u_j$ will be equal to zero. Then using the obtained expressions (7), (11) we can write the final integrated form of potential energy (37).

$$\Pi = \frac{EF}{6L}\left[7u_i^2 - 16u_i u_j + 2u_i u_k + 16u_j^2 - 16u_j u_k + 7u_k^2\right] - \frac{\alpha EF}{6}\left[-3T_i u_i + 4T_i u_j - T_i u_k - 4T_j u_i + 4T_j u_k + \right.$$
$$\left. + T_k u_i - 4T_k u_j + 3T_k u_k\right]. \quad (12)$$

Now, given that the nodal temperature values are known, we will minimize the potential energy by the nodal displacement values. As a result, we obtain a system of three linear algebraic equations

$$\left.\begin{aligned}
&1)\frac{\partial \Pi}{\partial u_i}=0;\Rightarrow \frac{7EF}{3L}u_i-\frac{8EF}{3L}u_j+\frac{EF}{3L}u_k=-\frac{\alpha EF}{2}T_i-\frac{2\alpha EF}{3}T_j+\frac{\alpha EF}{6}T_k;\\
&2)\frac{\partial \Pi}{\partial u_j}=0;\Rightarrow -\frac{8EF}{3L}u_i+\frac{16EF}{3L}u_j-\frac{8EF}{3L}u_k=\frac{2\alpha EF}{3}T_i-\frac{2\alpha EF}{3}T_k;\\
&3)\frac{\partial \Pi}{\partial u_i}=0;\Rightarrow \frac{EF}{3L}u_i-\frac{8EF}{3L}u_j+\frac{7EF}{3L}u_k=-\frac{\alpha EF}{6}T_i-\frac{2\alpha EF}{3}T_j+\frac{\alpha EF}{2}T_k.
\end{aligned}\right\} \quad (13)$$

Discretizing the considered rod by one quadratic finite element and taking, $q(x)=\frac{160}{\ell^2}x^2-\frac{160}{\ell}x$ we obtain the numerical results that are given in Table 1, and the corresponding distributions of the displacement field are given in Figure 2

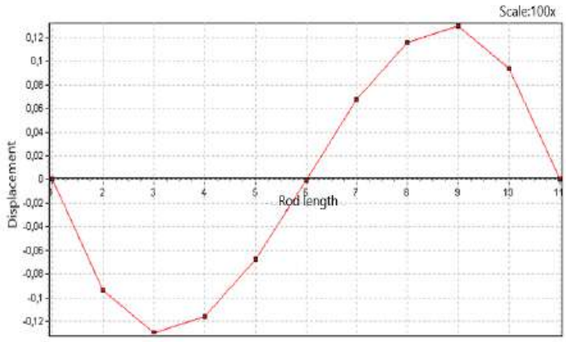
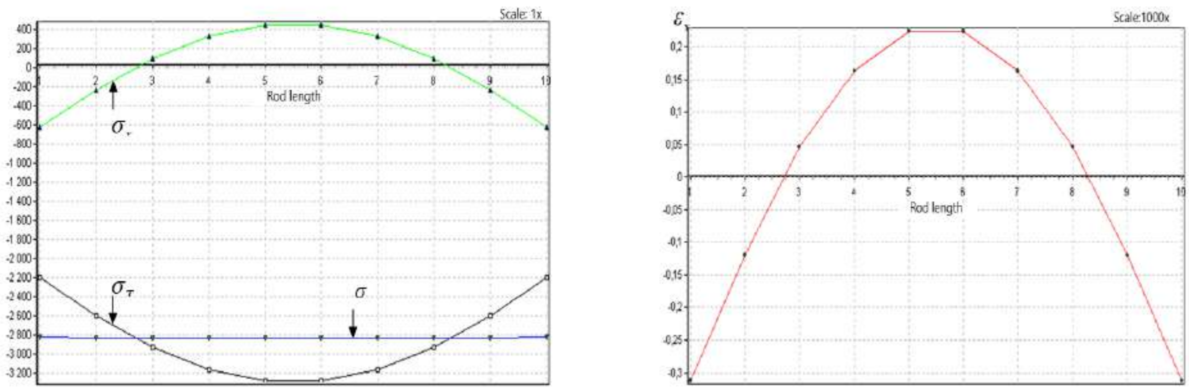


Figure 2 Distribution of the displacement field

It is evident from this figure 2 that due to the symmetry of the heat flow supplied to the lateral surface, the displacement field is also symmetrical. It is also observed that the points of the left half of the rod move against the direction of the coordinate axis, while the points of the right half move in the direction. In addition, since both ends of the rod are rigidly clamped and the heat flow supplied to the lateral surface is symmetrical relative to the middle point of the rod, the displacement values of the two extreme and middle points will be equal to zero. The distribution of the deformation and stress fields are shown in4Figure 3a), b) and Figure 4.



a) b)

Figure 3 Distribution fields $\varepsilon_x, \sigma_x, \sigma_T, \sigma$

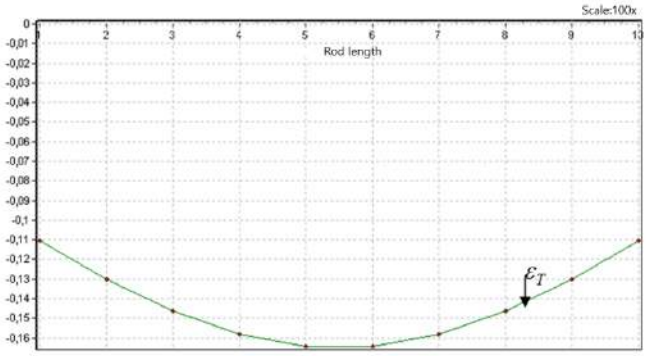


Figure 4. Distribution fields ε_T

According to Figure 3, the maximum value of elastic deformation, which is of a compressive nature, is achieved near the pinch point, and the greatest value of tensile elastic deformation is achieved near the middle of the point. In this case, the thermoelastic stress is of a compressive nature, and its average value over the entire rod is *–2836,8125 (kG/cm²)*. This value numerically exceeds the theoretical value by *3,4 (kG/cm²)*. It is also evident from Figure 3 that the nature of the elastic stress near the pinch point will be compressive and equal to σ_x = *–625,333 (kG/cm²)*, and at other points it is of a tensile nature. At that time, near the middle of the rod, the greatest tensile elastic stress will be *448 (kG/cm²)*, and the nature of the temperature stress over the entire rod will be compressive and equal to σ_x = *–2207,3125 (kG/cm²)*. Its greatest value is achieved in the middle of the rod. Where near the middle of the rod σ_x = *–3268,48 (kG/cm²)*. The nature of thermoelastic stress on average $\sigma = \sigma_x + \sigma_T =$ *–2836,8125 (kG/cm²)*.

Now we discretize the considered rod by two quadratic finite elements. In this case, the number of resolving equations is five. The length of each finite element is *15(cm)*. The numerical solutions obtained in twenty fixed sections of the rod are given in Table 2. This table gives the values of the strain and stress components. The results obtained show that the values of the thermoelastic stress in all sections of the rod fluctuate around. This result differs from the approximately analytical solution by only *0,8 (kG/cm²)*. Figure 5 shows the distribution of the displacement field along the length of the rod.

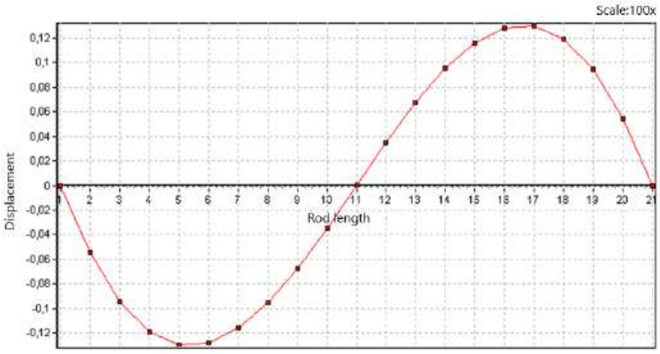
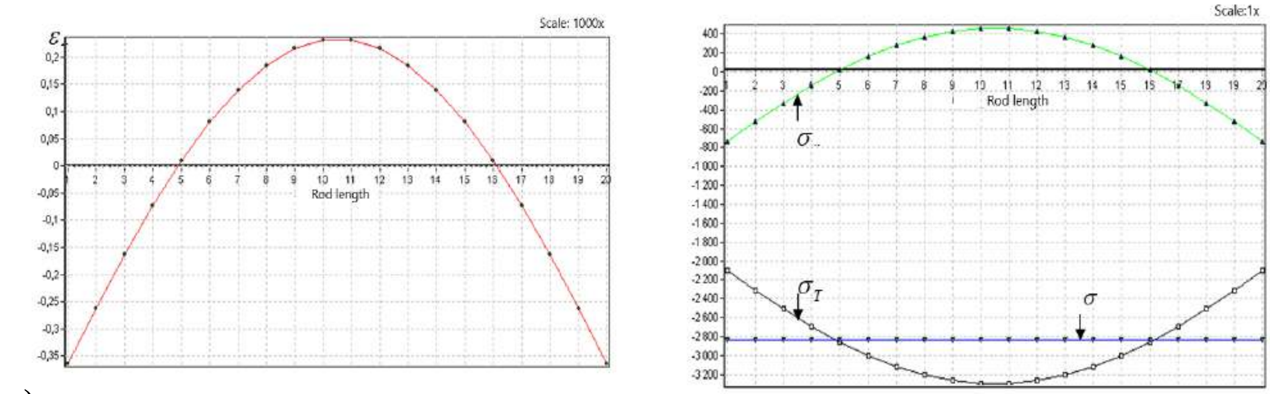


Figure 5 Distribution of the displacement field

From this figure 5 it is clear that the amplitude of the displacement of point 5, which corresponds to the displacement of the point with coordinate *7,5 (cm²)*, will be u_5=*–3268,48* (*cm*). At the same time, the displacement of the point with coordinate symmetrical to this point relative to the center of the rod *x = 22,5(cm)* will be u_{17}= *–0,01296* (*cm*). From this figure 5 it is clearly seen that the symmetry of the obtained numerical results is ensured. As a result, the displacement of the middle point of the rod with coordinate *x = 15 (cm)* u_{11}= 0 (*cm*). Figure 6a), b) and Figure 7 show the distribution of the deformation and stress field.



a) b)

Figure 6. Distribution fields $\varepsilon_x, \sigma_x, \sigma_T, \sigma$

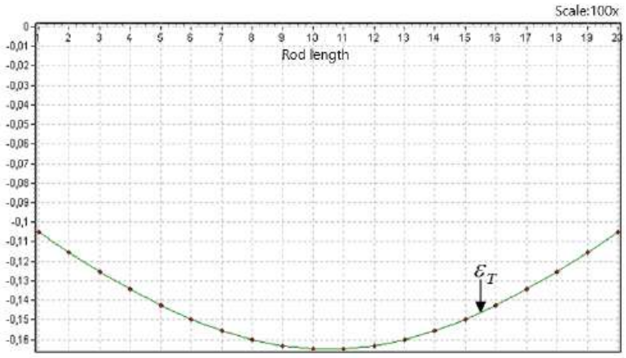


Figure 7 Distribution fields ε_T

From these results it is also evident that the symmetry of the problem is ensured.

Now we discretize the considered rod by ten quadratic finite elements of the same length ℓ = *(3cm).* In this case, the values of the thermoelastic stress will be σ_T *= –2833,37 (kG/cm²)*, and this exactly coincides with the results of the approximate analytical solution. The obtained values of the strain and stress components in fixed sections are given in Table 3, and the corresponding distribution of the displacement field is given in Figure 8.

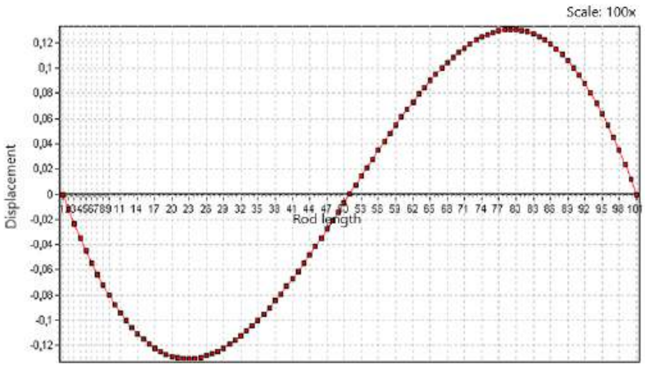
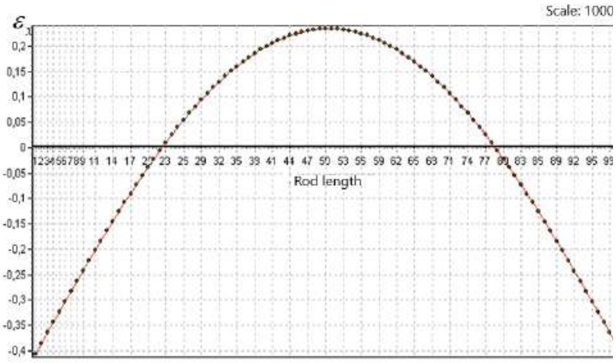
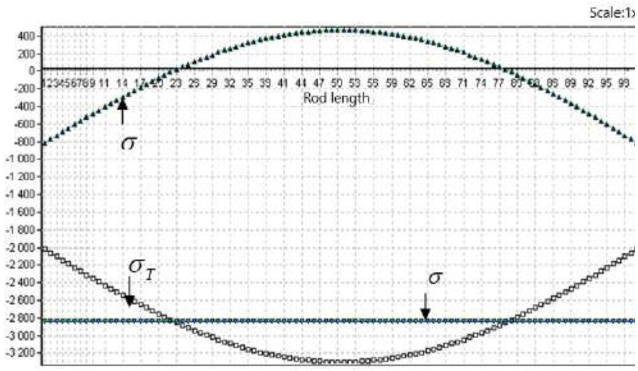


Figure 8 Distribution of the displacement field

The amplitude of the displacements of the midpoint of the first half of the rod will be u_{23}=*–0,0013* (*cm*), and average of the second half will be u_{79}=*–0,0013*(*cm*). From this figure 9a), b) it is clear that the symmetry of the problem is ensured, therefore the displacement of the midpoint of the rod u_{51}=*0* (*cm*). Figure 8 shows the law of distribution of the deformation and stress field.



a)



b)

Figure 9 Distribution fields $\varepsilon_x, \sigma_x, \sigma_T, \sigma$.

These graphs also demonstrate the symmetry of the problem in terms of parameters. The value of the greatest tensile elastic deformation $\mathcal{E}_x = 0,00023427$ and it occurs in the middle of the rod. Figure 10 shows the law of distribution of the temperature deformation field.

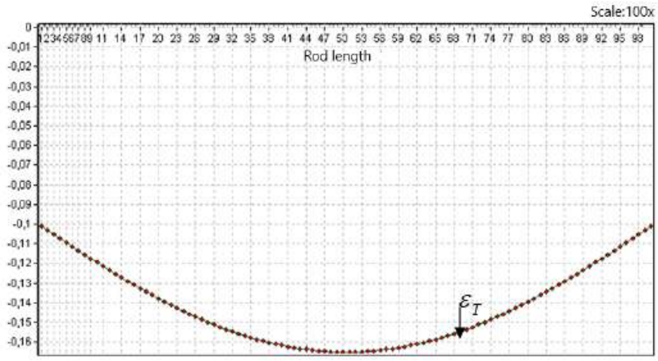


Figure 10. Distribution fields ε_T.

Results of the study

Thus, when solving the problem of thermal strength of structural elements exposed to a parabolic heat flow, it is desirable to discretize the considered rod by at least ten quadratic finite elements. However, it should be noted that the numerical results obtained even when discretizing the considered rod by one quadratic finite element will always be useful for engineering calculations. Because in this case the error is no more than 0.194%. But such an error is negligible in engineering calculations when it comes to stresses that do not exceed their permissible values.

Discussion

Now we will numerically investigate the influence of the length, heat transfer coefficient and ambient temperature on the thermal stress-strain state of a rod clamped at both ends in the presence of a heat flux on the lateral surface that changes according to a parabolic law of the following form

$$q(x) = \frac{160}{\ell^2}x^2 - \frac{160}{\ell}x.$$

We vary the value of the rod length $\ell,(см)$. Table 4 shows the value of the compressive force $R,(кГ)$ and stress $\sigma,(кГ/см^2)$ for six different values of the rod length.

Table 1. The influence of the rod length value on the thermal stress-strain state of the studied rod

№	$\ell,(см)$	$R,(кГ)$	$\sigma,(кГ/см^2)$	%
1.	30	-35586,66	-2833,33	100
2.	27	-32342	-2575	90,88
3.	24	-29306,66	-2333,33	82,35
4.	21	-26480,66	-2108,33	74,41
5.	18	-23864	-1900	67,06
6.	15	-21456,66	-1708,33	60,29

This table shows that the values of compressive force and stress decrease in a non-linear manner with decreasing rod length. The decrease in these values is motivated by the decrease in the area to which the heat flow is supplied. In addition, these results show that it is possible to regulate the value of compressive force and stress using the length of this rod element.

Now we numerically investigate the influence of the heat exchange coefficient value $h,(Bm/см^2 \cdot {}^\circ C)$ between the cross-sectional areas and the environments surrounding them. The results are given in Table 5.

Table 2. The influence of the heat transfer coefficient value on the thermal stress-strain state of the studied rod

№	$h,(Bm/см^2 \cdot {}^\circ C)$	$R,(kG)$	$\sigma,(кГ/см^2)$	%
1.	10	-35586,66	-2833,33	100
2.	9	-36982,22	-2944,44	103,92
3.	8	-38726,66	-3083,33	108,82
4.	7	-40969,52	-3261,9	115,12
5.	6	-43960	-3500	123,53
6.	5	-48146,66	-3833,33	135,29

This table shows that with a decrease in the value of the heat exchange coefficient, an increase in the values of the compressive force and stress is observed. In this case, this dependence occurs in a relatively nonlinear manner. The increase in values and with a decrease in value h is motivated by the fact that at low values h a small amount of heat is transferred from the rod to the environment.
The influence of the temperature surrounding the cross-sectional areas of the clamped ends of the rod $T_{OC},({}^\circ C)$ on values of compressing force R and σ a stress. The results of these studies are given in Table 6.

Table 3. Influence of the ambient temperature on the thermal stress-strain state of the rod under study

№	$T_{OC},({}^\circ C)$	$R,(kG)$	$\sigma,(кГ/см^2)$	%
1.	40	-35586,66	-2833,33	100
2.	35	-34016,66	-2708,33	95,59
3.	30	-32446,66	-2583,33	91,17
4.	25	-30876,66	-2458,33	86,76
5.	20	-29306,66	-2333,33	82,353

From this table 3 it is evident that with decreasing ambient temperature T_{OC}, the values R and σ decrease proportionally, but nonlinearly. This process is motivated by the fact that at low ambient temperatures a large amount of heat leaves the rod into the environment, i.e. the rod will cool relatively. This phenomenon can be used to control the value of the compressive force and stress using the temperature surrounding the cross-sectional areas of the clamped ends of the rod of the environment.

Conclusion

A mathematical model and numerical methods for the thermoelastic state of the rod were constructed pinched at two ends in the presence of heat flow on it surface of a surface varying in coordinate square law. It was revealed that when a heat flux with a parabolic variation is connected to the side surface, an increase in the length of the rod leads to an increase in the elongation of the rod. Thus, with a decrease in the length of the rod, the deformed state of the rod is maintained while maintaining a heat flow on it, changing by the quadratic law of attraction. In some cases, the value of the heat transfer coefficient, on the contrary, increases the crisis-deformed state. When measuring the ambient temperature, and also in some cases, the crisis-deformed state decreases.

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