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DEVELOPMENT OF STUDENTS' RESEARCH COMPETENCIES WHEN STUDYING ANALYTICAL AND PROJECTIVE GEOMETRY

Abstract

The article discusses the methods of developing research skills among future teachers of mathematics and physics in the process of studying analytical and projective geometry at the university using interactive learning tools. Let us note that research competencies are a component of the professional competencies of bachelors in the areas of training “Mathematics” and “Mathematics and Physics”. Therefore, in the course of the study, we developed and tested the educational and methodological complex “Analytical and Projective Geometry” using interactive programs, and also proposed methods, forms and criteria for assessing the research abilities of future mathematics and physics teachers. Methods of problem-based learning, practice-oriented learning, methods of analysis and synthesis, and observation methods were used. The relevance of the research topic lies in the problem of developing the research skills of future mathematics and physics teachers, and their use of such interactive programs as living geometry and geogeography can be used by them to create a research lesson. The methodological approach to teaching analytical and projective geometry presented in the article was implemented in the educational process in the course of “Analytical and projective geometry” in Pavlodar Pedagogical University named after A.Margulan during training of future bachelors of “Mathematics”, “Mathematics and Physics” and allowed to achieve positive results in teaching.

Keywords: analytical and projective geometry, research ability, development of research competence, future teacher, GeoGebra.

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АНАЛИТИКАЛЫҚ ЖӘНЕ ПРОЕКТИВТІК ГЕОМЕТРИЯНЫ ОҚЫТУ БАРЫСЫНДА СТУДЕНТТЕРДІҢ ЗЕРТТЕУ ҚҰЗЫРЕТТІЛЕРІН ДАМЫТУ

Аңдатпа

Мақалада интерактивті оқыту құралдарын пайдалана отырып, университетте аналитикалық және проективтік геометрияны оқу барысында болашақ математика және физика мұғалімдерінің зерттеушілік дағдыларын дамыту әдістері қарастырылған. Ғылыми-зерттеу құзыреттіліктері «Математика» және «Математика және физика» мамандықтары бойынша бакалаврлардың кәсіби құзыреттіліктерінің құрамдас бөлігі болып табылатынын атап өтейік. Сондықтан зерттеу барысында біз интерактивті бағдарламалар арқылы «Аналитикалық және проекциялық геометрия» оқу-әдістемелік кешенін және болашақ математика және физика мұғалімдерінің зерттеушілік қабілеттерін бағалаудың ұсынылған әдістерін, формалары мен критерийлерін әзірлеп, сынақтан өткіздік. Проблемалық оқыту әдістері, тәжірибеге бағытталған оқыту, талдау және синтез әдістері, бақылау әдістері қолданылды. Зерттеу тақырыбының өзектілігі болашақ математика және физика мұғалімдерінің зерттеушілік дағдыларын дамыту мәселесінде және тірі геометрия және геогебра сияқты интерактивті бағдарламаларды қолдану арқылы сабақ – зерттеу құруға болады. Мақалада келтірілген аналитикалық және проективтік геометрияны оқытудың әдістемелік тәсілі Марғұлан университетінде «Аналитикалық және проективтік геометрия» курсын оқу үдерісінде «Математика», «Математика және физика» бағдарламалары бойынша болашақ бакалаврларды дайындауда жүзеге асырылды және оң нәтижелерге қол жеткізуге мүмкіндік берді.

Түйін сөздер: аналитикалық және проективтік геометрия, зерттеушілік қабілеті, зертттеу құзыреттілігін қалыптастыру, болашақ мұғалім, геогебра.

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РАЗВИТИЕ ИССЛЕДОВАТЕЛЬСКИХ КОМПЕТЕНЦИЙ СТУДЕНТОВ ПРИ ИЗУЧЕНИИ АНАЛИТИЧЕСКОЙ И ПРОЕКТИВНОЙ ГЕОМЕТРИИ

Аннотация

В статье рассматриваются способы организации учебно-исследовательской деятельности студентов и методы развития исследовательских умений будущих учителей математики и физики в процессе изучения аналитической и проективной геометрии в вузе. Отметим, что исследовательские компетенции являются составляющей профессиональных компетенций бакалавров по направлению подготовки «Математика» и «Математика и физика». Поэтому в ходе исследования нами был разработан и апробирован учебно-методический комплекс «Аналитическая и проективная геометрия» с использованием интерактивных программ, предложены методы, формы, критерии оценки исследовательских умений будущих учителей математики и физики. Применялись методы проблемного обучения, практико-ориентированного обучения, методы анализа и синтеза, методы наблюдения. Актуальность темы исследования заключается в проблеме развития исследовательских умений будущих учителей математики и физики, а использование таких интерактивных программ как живая геометрия, геогебра могут быть использованы ими для составления урока - исследования. Представленный в статье методический подход к обучению аналитической и проективной геометрии был реализован в учебном процессе на занятиях по курсу «Аналитическая и проективная геометрия» в Павлодарском педагогическом университете имени А.Маргулана при подготовке будущих бакалавриатов по направлениям «Математика», «Математика и физика» и позволил добиться положительных результатов в обучении.

Ключевые слова: аналитическая и проективная геометрия, исследовательская способность, формирование исследовательской компетенции, будущий учитель, геогебра.

Introduction

The relevance of the topic of the article is justified by the modern requirement for the training of future teachers of mathematics and physics, namely the ability to conduct research activities at school. The purpose and objectives of the lesson are to develop research skills in lessons on analytical and projective geometry using an interactive program. The methodology proposed in the article is based on the need to use such programs as living geometry and geoalgebra in practical and independent classes. The main conclusions are the need to develop research competencies, develop modern teaching tools and improve practical training for effective work. The importance of emphasizing the use of interactive programs for solving and proving geometric problems is also emphasized, which contributes to the development of research competencies of future mathematics and physics teachers.

Due to transition to a credit-unit system of education, projective geometry is not a separate discipline, it is as one of the modules of the discipline “Analytical and projective geometry”. Therefore, to avoid deterioration in quality of geometric training of students of physics and mathematics direction we use computers, interactive means of support in the study of the discipline. Note that by the concept of “ability to use interactive teaching tools” we mean the ability to use ready-made programs for conducting experiments and solving problems on a plane and in space such as “Living Geometry” [1] and GeoGebra [2], “Cabri Geometry”, “C.a.R.”, “GeoNext”, “Mathematical Constructor”. Among these interactive aids we often used the GeoGebra program, because unlike other programs for dynamic manipulation of geometric objects, the idea of the GeoGebra environment lies on an interactive combination of geometric, algebraic and numeric representation. GeoGebra is a free program that provides the ability to create geometrical constructions. The use of computer technology in the geometry course during training of a teacher of mathematics and physics can significantly improve the quality of teaching material absorption. In addition, it sets a certain direction of students’ mathematical training.

Analyzing domestic and foreign experience in using a computer as one of the means of teaching geometry, we can conclude that universities and pedagogical institutions have accumulated certain experience and obtained profound results that have theoretical and practical significance in the field of computerization of geometry [3-8]. Abroad, two software products of dynamic geometry enjoy the greatest popularity in the process of geometric training of students. One of them (Cabri geometry) is developed in France (J.-M. Laborde, F. Bellemain, etc.), the other (Geometer's Sketchpad) - in the USA (J. King, D. Sher, etc.).

The most complete and interesting results in the field of computerization of the basic geometry course, supported by a large number of publications and implementation in the educational process [9-10].

Materials and methods

We will begin the description of the educational and methodological complex we have developed with the course of lectures we have developed "Analytical and Projective Geometry. It consists of thirty lectures on major additional topics; examples of standard and non-standard tasks for the development of creative thinking; problem questions are focused on the ability to perform research actions. The difficulty of studying this discipline is that students have been studying Euclidean geometry all their conscious life and are used to its standards. Therefore, when studying projective geometry, it becomes difficult for them to understand its structure and basic elements.

When preparing a lecture and delivering it, problem-based teaching methods are used. Experience shows that problem lectures are perceived by students with interest and understanding [11]. Therefore, the use of problem situations in lectures can promote a creative approach to problem solving. Tasks should be structured in such a way that students can propose their own ways to solve the problem. Let's look at some projective geometry questions that require creativity. For example, the intersection of parallel lines. From the school course of geometry students know that parallel lines are lines that do not have a common point. In reality, it turns out that Euclidean geometry is not the only option of this science, that there are non-Euclidean types of geometry such as Lobachevsky geometry, spherical, projective, etc.

Table 1. Structure of a problem lecture on projective geometry on the example of the topic "Desargues Theorem".

Stages of the lecture	*Teacher's objective*	*Techniques and methods of teacher's activity*
Introduction	*Cultivate student interest in the topic and capture the attention of the audience*	*Task: parallel lines do not intersect. Why do we see train tracks crossing in the distance at one point?*
Setting of a problem	*Determine the goal, objectives of the problem, show relevance, find out where contradictions appear*	*Projective geometry radically extends the well-known duality principle. Compare. Students should gain a deeper understanding of the notion of an incidence relation, or the belonging of one subspace to another. Realize that in projective geometry, lines are treated as points and three-dimensional spaces as lines. They then discover an unexpected connection between the generalized Desargues theorem and the triangle altitude theorem.*
Break a big problem into small tasks	*Clearly identify key issues and create an action plan.*	*The study of the Desargues' configuration in various options and modifications, contributes to a better understanding of perspective theory.* *Give examples on the use of Desargues' generalized theorem. Students give examples of use in architecture, design, fine arts, etc.*
Explanation of your solution, method, method	*Compare your solution method with others and analyze*	*It is reasonable to demonstrate that the Desargues configuration is not only fundamental to projective geometry, being the basic configuration of*

		projection and perspective correspondence of series of points and lines, but also has very numerous applications in architecture and design. Students justify evidence, judgments, arguments and use methods of critical analysis and comparison
Conclusions. Analysis	*Analyze, draw a conclusion*	*However, it should be taken into account that the theorem is quite complex, therefore, to master it, the department uses computer programs that allow one to become familiar with all the basic invariant properties of the Desargues configuration.*

Thus, the use of a problematic presentation of lecture material when studying the course “Analytical and Computational Geometry” contributes to the formation of deep knowledge among students in combination with the development of interest and motivational aspects in the learning process, and also contributes to the intensification of students’ research activities. Therefore, it is perceived with interest by students, shows the creative abilities of students, which, in turn, contributes to a deep understanding of the topic and is an effective way of learning.

In practical classes there is an exchange of knowledge, ideas between students, students and instructor. To improve research skills, we used interactive programs in practical classes. For example, the study of analytical geometry at a pedagogical university traditionally ends with the topic “surface of the second degree.” This topic has great potential for demonstrating the process of exploring the geometric properties of surfaces using their algebraic equations. Modern digital technologies make it possible to simulate various geometric structures and show them from convenient angles. For this purpose, it is proposed to use the GeoGebra program, which successfully combines the ability to depict three-dimensional figures and perform complex mathematical calculations. In the section of a conic surface with a plane, an ellipse, a hyperbola and a parabola were obtained, and the type of the section and the name of the curve changed depending on the angle of inclination of the cutting plane. In addition, the possibilities of the use of GeoGebra program were considered in all sections of the theory of the second-degree surface: the method of sections for the study of the surface configuration; surfaces of rotation; cylindrical and conical surfaces; ellipsoid, hyperboloids and paraboloids; rectilinear formations of one-sheeted hyperboloid and hyperboloid paraboloid. Special attention was paid to the problem of classifying surfaces by simplifying their general equation when moving to a new coordinate system. With the help of GeoGebra it was possible to visualize the dynamics of such coordinates transformation, presenting it as a result of the corresponding space motion.

For example, when covering topics about surfaces of revolution, single-sheeted, double-sheeted hyperboloid, hyperbolic paraboloid, the use of GeoGebra made it possible not only to see when rotating, but also to look when cutting the surface, which made it possible to use it in solving problems (Figure 1).

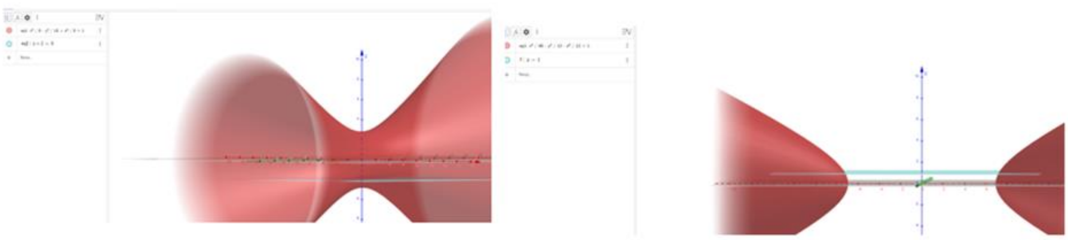

Figure 1. Visualization of surfaces of revolution and hyperbolic surfaces using GeoGebra

Projective geometry is a rather difficult section of geometry for students to understand. As experience shows, creating videos with solutions to problems and demonstrations of proofs of theorems contributes to a faster and deeper understanding of the subject and the emergence of interest in its study. GeoGebra just presents such opportunities, namely, the possibility of step-by-step viewing of the solution of a problem or proof of a theorem. Therefore, the computer program GeoGebra was used in practical classes to solve problems in analytical and projective geometry. For example, when studying the theorem of Desargues which states: On the projective plane there are six different points C, B, D, E, F, G. The lines BE, DF, CG intersect in one point A if and only if the intersection points of pairs of lines CB and EG, BD and EF, CD and GF lie on the same line. Students prove this theorem using the GeoGebra program and obtain the following image (Figure 1). By moving the triangles students see that the three points are always collinear.

Let us emphasize to students that in projective geometry instead of the words "a point lies on a line" and "a line passes through a point" we can use the words "a point and a line are incident". If we now write down the formulation of the theorem using this artificial turn, then in the resulting text the words "point" and "line" can be interchanged. As a result of this "linguistic" procedure, Desargues' direct theorem can be turned into an inverse theorem and written as: If the points of intersection of lines CB and EG, BD and EF, CD and GF are collinear, then lines BE, DF, CG intersect at the same point A. . Students prove this theorem using the GeoGebra program and receive the following image (Figure 2). By moving the triangles, students see that the three points are always collinear.

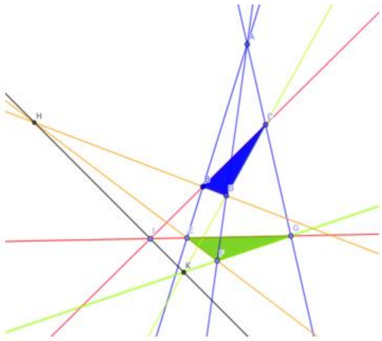

Figure 2. GeoGebra illustration of Desargues' theorem showing collinear intersection points

Students begin to realize that in projective geometry the same "transformation" can be applied to the text of any theorem. After all, in the projective plane, unlike the Euclidean plane, there are no parallel lines. That is, "any two lines are incident to one common point", "any two points are incident to one common line". Since there are no distances, angles, or areas in projective geometry, all theorems refer only to the incidence of points and lines. The condition and conclusion involve mainly collinear triples of points (three points are incident to one line) and competitive triples of lines (three lines are incident to one point).

Thus, if a theorem of projective geometry is proved, its dual theorem, which is obtained from it by swapping points and lines, can also be considered as proved. As an important example, one tries to construct a theorem dual to the theorem about the projective mapping of one line to another.

While passing the topics on projective geometry students showed their creative and research abilities by applying linear perspective, shadow theory, Desargue configurations in painting, design, architecture. Therefore, for independent work, students were given tasks to study the Desargues configuration in different versions and modifications, the theory of perspective and shadow (Figure 3). We learned to use a special type of perspective art - anamorphosis (Figure 4).

However, it should be taken into account that many provisions of the theory are quite complicated and it takes considerable time to master it.

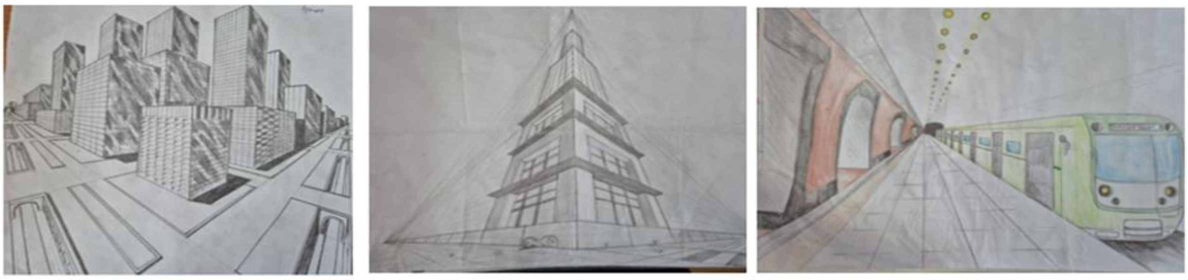

Figure 3. Students' independent work on Desargues configurations and perspective and shadow theory using GeoGebra

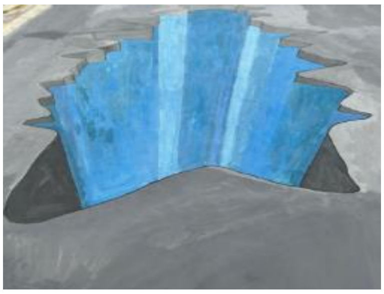

Figure 4. Example of applying anamorphosis as a special type of perspective art

By the end of the semester, students knew the basic methods of constructing perspective images of geometric figures; the basic rules of applying perspective projections in drawing and analyzing perspective images; solved the problems of constructing shadows of objects in perspective at different positions of the light source

Results and discussion

When studying the discipline "Analytical and Projective Geometry" with second-year students at Margulan University, we used pre-developed indicators and test materials to diagnose the level of development of the research abilities of future mathematics and physics teachers [11]. During the study, we used the levels we developed: high, sufficient, threshold, low, critical, which made the assessment more objective [11]. During our study, students were not divided into control and experimental groups. We carried out diagnostics at the beginning of using the educational and methodological complex and after. The criteria are shown in Table 2.

Table 2. Criteria of the level of research skills formation

Criterion	*Level and score of research skills formation*				
	Critical /1	*Low /2*	*Cut-off /3*	*Sufficient /4*	*High /5*
A. Ability to see the problem	*No interest shown*	*Delves into the problem as needed*	*Delves into as needed*	*The problem is viewed one-sidedly*	*Able to research a problem from different angles*
B. Ability to hypothesize	*No interest shown*	*Doesn't know how to hypothesize*	*Produces a hypothesis during teamwork*	*Can reason logically*	*Able to hypothesize not only through logical reasoning, but also as into-itive thinking*
C. Ability to set a problem and propose ways to solve it	*No interest shown*	*Doesn't know how to put forward a task*	*Sets goals, solves problems together as a team*	*Knows how to set a problem but has*	*Able to independently set solutions and find ways to solve it*

				difficulty in solving it	
D. Ability to analyze	*No interest shown*	*Doesn't know how to analyze*	*Makes analysis when working with a group*	*Able to analyze, has difficulty in drawing conclusions*	*Able to analyze results, draw conclusions*
E. Ability to make judgments	*No interest shown*	*Does not know how to speak on the topic*	*Makes his judgments with uncertainty*	*Makes incomplete statements*	*Can not only make judgments but also draw inferences*
F. Ability to observe	*No interest shown*	*Can't repeat during observations*	*Observes and understands problem solving*	*Draws conclusions from his observations*	*Not only knows how to observe, but also draws his conclusions based on those observations*
G. Ability to use interactive programs	*No interest shown*	*Does not know how to use*	*Uses with the teacher*	*Uses in some cases*	*Uses interactive programs in problem solving*
H. Classification skills	*No interest shown*	*Does not know how to divide by common features*	*Only when working as a team*	*Able to classify by common features*	*Able not only to classify by general features, by the course of decision, etc., but also to distinguish common es-sential features in them*

At the beginning of the semester, students showed low indicators, i.e., they were not able to find and work with information, with literature, did not show persistence in solving problems, were not able to analyze and correctly formulate the solution and the main points of the research, etc.

Taking into account that the maximum score is 5 and the minimum score is 1 score, we converted the obtained indicators into scores

$$X = \frac{A + B + C + D + E + F + G + H}{8}$$

Subsequently, analyzing the results at the initial stage of the study, where the level of development of research skills was low, we saw that threshold and low scores predominated. The level of development of the students' research skills with the use of the same methodology was repeatedly diagnosed at the final stage. Let's conduct a statistical analysis using the Wilcoxon T-test and determine whether there is an intensive shift in indicators in this direction (Table 3).

H_0: Post-experiment scores are higher than pre-experiment scores. H_1: Post-experiment scores are less than pre-experiment scores.

The sum of the ranks column is equal to ∑=666.

Checking the correctness of the matrix based on checksum calculation:

$$\sum x_{ij} = \frac{(1+n)n}{2} = \frac{(1+36)36}{2} = 666$$

Table 3. Application of the Wilcoxon T-test to determine the reliability of the shift in indicators of the level of development of students' research skills and abilities

№	Before measurement, t_{before}	After measurement, t_{after}	Difference (t_{before}-t_{after})	The absolute value of the difference	Difference rank number
1.	2.8	2.8	0	0	8.5
2.	2.1	2.2	0.1	0.1	26
3.	3.2	3.2	0	0	8.5
4.	2.6	2.7	0.1	0.1	26
5.	3.1	3.2	0.1	0.1	26
6.	4.1	4.2	0.1	0.1	34.5
7.	2.3	2.3	0	0	8.5
8.	2.4	2.5	0.1	0.1	26
9.	3.1	3.1	0	0	8.5
10.	3.2	3.3	0.1	0.1	17.5
11.	4.2	4.2	0	0	8.5
12.	2.8	2.8	0	0	8.5
13.	2.8	2.9	0.1	0.1	26
14.	3.1	3.2	0.1	0.1	26
15.	3.2	3.2	0	0	8.5
16.	4.1	4.2	0.1	0.1	34.5
17.	3.1	3.1	0	0	8.5
18.	2.0	2.1	0.1	0.1	26
19.	3.1	3.2	0.1	0.1	26
20.	2.6	2.5	-0.1	0.1	26
21.	2.2	2.4	0.2	0.2	36
22.	2.3	2.4	0.1	0.1	26
23.	3.1	3.1	0	0	8.5
24.	3.2	3.2	0	0	8.5
25.	2.6	2.7	0.1	0.1	26
26.	2.6	2.5	-0.1	0.1	26
27.	3.1	3.2	0.1	0.1	26
28.	3.2	3.2	0	0	8.5
29.	4.0	4.1	0.1	0.1	17.5
30.	4.6	4.6	0	0	8.5
31.	2.5	2.6	0.1	0.1	26
32.	2.5	2.5	0	0	8.5
33.	2.1	2.1	0	0	8.5
34.	3.3	3.3	0	0	8.5
35.	3.4	3.4	0	0	8.5
36.	3.4	3.5	0.1	0.1	26
	Sum				666

The column sum and the checksum are equal, so the ranking is correct. Now let us mark the directions that are atypical, in this case, negative. The sum of the ranks of these "rare" directions is the empirical value of the criterion T: $T=\sum Rt=26+26=52$

We use the table to find the critical values for the Wilcoxon T-criterion for n=36: $T_{cr}=185$ ($p\leq 0.01$). $T_{cr}=227$ ($p\leq 0.05$). The zone of significance in this case extends to the left; indeed, if there were no "rare", in this case positive, directions at all, then the sum of their ranks would be zero. In this case, however, the empirical value of T falls into the zone of significance: $T_{emp}<T_{cr}(0.01)$. Hypothesis H_0 is accepted. Post-experiment indicators exceed the values of pre-experiment indicators.

A statistically significant positive shift was noted: indicators of the level of development of research skills at the final stage increased compared to similar indicators at the first defining stage.

The study of the formation of research skills and abilities when studying the module of the discipline “Analytical and Projective Geometry” showed positive statistics. Thus, the results obtained from a small sample are reliable. They can be extended to the entire population of subjects and contain methodological recommendations.

Conclusion

The analysis of the results of our research on the problem of formation of students’ research abilities allows us to talk about the significance and feasibility of using interactive programs, in particular, the dynamic system GeoGebra, which favorably affects the students’ understanding of the educational material in the study of the discipline “Analytical and projective geometry”. The level of learning on the topic is not reduced if you use computer dynamic drawings instead of paper textbooks and notebooks. This does not mean that it is necessary to give up writing, but partial replacement of manual writing with computerized drawings takes place.

This study is aimed at ensuring the purposeful mastering by future teachers of mathematics and physics of the theoretical foundations of research activity through the formation of their research skills of a generalized nature and at organizing special training of students to carry out research activities in geometry lessons.

Список использованных источников

[1] Orazali G., Dauletkulova A., Mekebayev N. Methodology of teaching geometry in the context of digitalization of education. Bulletin of Ablai Khan KazUIRandWL Series “Pedagogical Sciences” No 1 (76) 2025, pp. 446-457 DOI: https://doi.org/10.48371/PEDS.2025.76.1.029

[2] Clark-Wilson A., Robutti O., Thomas M. Teaching with digital technology. ZDM Mathematics Education. - 2020. - No52(7). - C. 1223–1242 DOI:10.1007/s11858-020-01196-0

[3] Yarullin, I. F., Bushmeleva, N. A., & Tsyrkun, I. I. The Research Competence Development of Students Trained In Mathematical Direction. International Electronic Journal of Mathematics Education, 2015, 10(3),c. 137-146. https://doi.org/10.29333/iejme/296

[4] Nazarovich, S. T., & Kurudirek, A. An Overview of Existing Problems in Teaching the Science ”Fundamentals of Geometry”. International Journal of Social Sciences & Educational Studies, 2024, 11(2), 91-106. https://doi.org/10.23918/ijsses.v11i2p91

[5] Wardhani I. S. et al. The Model of Geometry Learning With Spatial Skills Features: Is It Possible? //Journal of Higher Education Theory & Practice. 2023, T. 23, №. 14. DOI: https://doi.org/10.33423/jhetp.v23i14.6397

[6] Osta, Iman & Laborde, Collete & Hoyles, Celia & Jones, Keith & Graf, Klaus-D & Hodgson, Bernard. Computer technology and the teaching of geometry. 1998, DOI:10.1007/978-94-011-5226-6_5

[7] Hamidah H. et al. Self-investigation exercises: geometry learning strategies for prospective teacher students //Journal of Education and Learning (EduLearn). – 2025. – T. 19. – №. 2. – C. 814-830. DOI:10.11591/edulearn.v19i2.21012

[8] Matheou Argyrou Aphrodite, & Panaoura Rita. Young Students’ Ability on Understanding and Constructing Geometric Proofs . Social Education Research, 2021, 2(2), 121–133. https://doi.org/10.37256/ser.222021784

[9] Noviana A. P., Nurhayati E., Rustiana R. Developing Students' Mathematical Communication Skills on Geometric Number Patterns Through Group Investigation and Peer Teaching //International Journal of Geometry Research and Inventions in Education (Gradient). – 2024. – T. 1. – №. 2. – C. 94-101. DOI:10.56855/gradient.v1i2.1249

[10] Drushlyak M. G. et al. Methodology of formation of modeling skills based on a constructive approach (on the example of GeoGebra) //CTE Workshop Proceedings. – 2021. – T. 8. – C. 458-472. DOI: https://doi.org/10.55056/cte.300

[11] Zhunussova R.A., Seitkhanova A.K., Kissabekova A.A., Alina A.B.. Formation of students' research abilities while studying the discipline "Optics" // Recent Contributions to Physics, 86(3), 85–94. https://doi.org/10.26577/RCPh.2023.v86.i3.010

References

[1] Orazali G., Dauletkulova A., Mekebayev N. Methodology of teaching geometry in the context of digitalization of education. Bulletin of Ablai Khan KazUIRandWL Series "Pedagogical Sciences" No 1 (76) 2025, pp. 446-457 DOI: https://doi.org/10.48371/PEDS.2025.76.1.029

[2] Clark-Wilson A., Robutti O., Thomas M. Teaching with digital technology. ZDM Mathematics Education. - 2020. - No52(7). - C. 1223–1242 DOI:10.1007/s11858-020-01196-0

[3] Yarullin, I. F., Bushmeleva, N. A., & Tsyrkun, I. I. The Research Competence Development of Students Trained In Mathematical Direction. International Electronic Journal of Mathematics Education, 2015, 10(3),c. 137-146. https://doi.org/10.29333/iejme/296

[4] Nazarovich, S. T., & Kurudirek, A. An Overview of Existing Problems in Teaching the Science "Fundamentals of Geometry". International Journal of Social Sciences & Educational Studies, 2024, 11(2), 91-106. https://doi.org/10.23918/ijsses.v11i2p91

[5] Wardhani I. S. et al. The Model of Geometry Learning With Spatial Skills Features: Is It Possible? //Journal of Higher Education Theory & Practice. 2023, T. 23, №. 14. DOI: https://doi.org/10.33423/jhetp.v23i14.6397

[6] Osta, Iman & Laborde, Collete & Hoyles, Celia & Jones, Keith & Graf, Klaus-D & Hodgson, Bernard. Computer technology and the teaching of geometry. 1998, DOI:10.1007/978-94-011-5226-6_5

[7] Hamidah H. et al. Self-investigation exercises: geometry learning strategies for prospective teacher students //Journal of Education and Learning (EduLearn). – 2025. – T. 19. – №. 2. – C. 814-830. DOI:10.11591/edulearn.v19i2.21012

[8] Matheou Argyrou Aphrodite, & Panaoura Rita. Young Students' Ability on Understanding and Constructing Geometric Proofs. Social Education Research, 2021, 2(2), 121–133. https://doi.org/10.37256/ser.222021784

[9] Noviana A. P., Nurhayati E., Rustiana R. Developing Students' Mathematical Communication Skills on Geometric Number Patterns Through Group Investigation and Peer Teaching //International Journal of Geometry Research and Inventions in Education (Gradient). – 2024. – T. 1. – №. 2. – C. 94-101. DOI:10.56855/gradient.v1i2.1249

[10] Drushlyak M. G. et al. Methodology of formation of modeling skills based on a constructive approach (on the example of GeoGebra) //CTE Workshop Proceedings. – 2021. – T. 8. – C. 458-472. DOI: https://doi.org/10.55056/cte.300

[11] Zhunussova R.A., Seitkhanova A.K., Kissabekova A.A., Alina A.B.. Formation of students' research abilities while studying the discipline "Optics" // Recent Contributions to Physics, 86(3), 85–94. https://doi.org/10.26577/RCPh.2023.v86.i3.010