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THE PARAMETER IDENTIFICATION PROBLEM FOR SYSTEM OF DIFFERENTIAL EQUATIONS

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Abstract

In this paper, the parameter identification problem for system of ordinary differential equations is considered. The parameter identification problem for system of ordinary differential equations is investigated by the Dzhumabaev's parametrization method. At first, conditions for a unique solvability of the parameter identification problem for system of ordinary differential equations are obtained in the term of fundamental matrix of system's differential part. Further, we establish conditions for a unique solvability of the parameter identification problem for system of ordinary differential equations in the terms of initial data. Algorithm for finding of approximate solution to a unique solvability of the parameter identification problem for system of ordinary differential equations is proposed and the conditions for its convergence are setted. Results this paper can be use for investigating of various problems with parameter and control problems for system of ordinary differential equations. The approach in this paper can be apply to the parameter identification problems for partial differential equations.

Keywords: parameter identification problem, system of ordinary differential equations, Dzhumabaev's parametrization method, algorithm, parameter, unique solvability.

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ДИФФЕРЕНЦИАЛДЫҚ ТЕҢДЕУЛЕР ЖҮЙЕСІ ҮШІН ПАРАМЕТРДІ ИДЕНТИФИКАТТАУ ЕСЕБІ

Бұл жұмыста жәй дифференциалдық теңдеулер жүйесі үшін параметрді идентификаттау есебі қарастырылады. Жәй дифференциалдық теңдеулер жүйесі үшін параметрді идентификаттау есебі Джумабаевтың параметрлеу әдісімен зерттеледі. Алдымен, жәй дифференциалдық теңдеулер жүйесі үшін параметрді идентификаттау есебінің бірмәнді шешілімділік шарттары жүйенің дифференциалдық бөлігінің фундаменталдық матрицасы терминінде алынады. Одан кейін біз жәй дифференциалдық теңдеулер жүйесі үшін параметрді идентификаттау есебінің бірмәнді шешілімділік шарттарын бастапқы берілімдер терминінде тағайындаймыз. Жәй дифференциалдық теңдеулер жүйесі үшін параметрді идентификаттау есебінің жуық шешімін табу алгоритмі ұсынылады және оның жинақтылығы шарттары орнатылады. Осы мақаланың нәтижелері жәй дифференциалдық теңдеулер жүйесі үшін көптеген параметрі бар есептерді және басқару есептерін зерттеуге пайдалануға болады. Бұл мақалада ұсынылған әдіс дербес туындылы дифференциалдық теңдеулер үшін параметрді идентификаттау есептерінде қолданыс табуы мүмкін.

Түйін сөздер: параметрді идентификаттау есебі, жәй дифференциалдық теңдеулер жүйесі, Джумабаевтың параметрлеу әдісі, алгоритм, параметр, бірмәнді шешілімділік.

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ЗАДАЧА ИДЕНТИФИКАЦИИ ПАРАМЕТРА ДЛЯ СИСТЕМЫ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ

В данной работе рассматривается задача идентификации параметра для системы обыкновенных дифференциальных уравнений. Задача идентификации параметра для системы обыкновенных дифференциальных уравнений исследуется методом параметризации Джумабаева. Сначала, условия однозначной разрешимости задачи идентификации параметра для системы обыкновенных дифференциальных уравнений получены в терминах фундаментальной матрицы дифференциальной части системы. Затем мы устанавливаем условия однозначной разрешимости задачи идентификации параметра для системы обыкновенных дифференциальных уравнений в терминах исходных данных. Предлагается алгоритм нахождения приближенных решений задачи идентификации параметра для системы обыкновенных дифференциальных уравнений и устанавливаются условия его сходимости. Результаты этой работы могут быть использованы для исследования различных задач с параметром и задач управления для системы обыкновенных дифференциальных уравнений. Предлагаемый в данной работе метод может быть применен к исследованию задач идентификации параметра для дифференциальных уравнений в частных производных.

Ключевые слова: задача идентификации параметра, система обыкновенных дифференциальных уравнений, метод параметризации Джумабаева, алгоритм, параметр, однозначная разрешимость.

Introduction.

As well-known, the parameter identification problem arises in the various processes of economics and econometrics, when the value of one or more parameters in an economic model cannot be determined from observable variables. It is closely related to non-identifiability in statistics and econometrics, which occurs when a statistical model has more than one set of parameters that generate the same distribution of observations, meaning that multiple parameterizations are observationally equivalent.

The parameter identification problems of various type are studied by many authors (in [1-8] can be seen bibliography and overview). For solving these problems are used different methods and approaches.

Conditions of solvability, unique solvability and continuous dependence from the initial data are established in the different terms. Despite this, finding effective conditions for the solvability of the parameter identification problem for systems of differential equations remains open and relevant.

In the present paper we propose a new approach for the study and solving of the parameter identification problem for the system of differential equations based on the Dzhumabaev's parameterization method [9-20].

The method was originally offered in [9, 10] for solving two-point boundary value problems for a linear differential equation. The method consists in partition of the interval apart, the introduction of additional parameters and reducing the original problem to the equivalent multi-point boundary value problem with parameters. The algorithms for finding a solution to the equivalent boundary value problem are constructed. The conditions for convergence of the algorithms that ensure the unique solvability of two-point boundary value problem with parameter are obtained. Coefficient criteria for unique solvability of considered problem are established in the terms of some matrix composed by the initial data.

Statement of problem.

On the interval $[0,T]$ we consider identification parameter problem for system of differential equations of first order in the following form

$$\frac{dx}{dt} = A(t)x + B(t)\mu + f(t), \qquad t \in [0,T], \quad x \in R^n, \qquad \mu \in R^n, \tag{1}$$

$$x(0) = x_0, \tag{2}$$

$$x(T) = x_1, \tag{3}$$

where $x(t) = colon(x_1(t), x_2(t),..., x_n(t))$ is unknown vector function, μ is unknown vector, the $(n \times n)$ matrices $A(t)$, $B(t)$, and n vector function $f(t)$ are continuous on $[0,T]$, x_0, x_1 are constant vectors.

A pair $(x^*(t), \mu^*)$ is called *a solution* to problem (1)--(3), if:

i) the function $x^*(t)$ is continuous and continuously differentiable on $[0,T]$, $\mu^* \in R^n$;

ii) it satisfies of the following system

$$\frac{dx^*}{dt} = A(t)x^* + B(t)\mu^* + f(t);$$

iii) the following equalities are fulfilled: $x^*(0) = x_0$, $x^*(T) = x_1$.

For solve problem (1)--(3) we use a fundamental matrix of system

$$\frac{dx}{dt} = A(t)x \ . \tag{4}$$

Let $\Phi(t)$ be the fundamental matrix of system (4) satisfying of condition $\Phi(0) = I$, where I is unit matrix on dimension n.

Using initial condition (2) we can write a solution of system (1) in the following form

$$x(t) = \Phi(t)x_0 + \Phi(t)\int_0^t \Phi^{-1}(\tau)B(\tau)d\tau\mu + \Phi(t)\int_0^t \Phi^{-1}(\tau)f(\tau)d\tau . \tag{5}$$

Further, finding value of function $x(t)$ for $t = T$ and substituting it into condition (3), we obtain

$$x(T) = \Phi(T)x_0 + \Phi(T)\int_0^T \Phi^{-1}(\tau)B(\tau)d\tau\mu + \Phi(T)\int_0^T \Phi^{-1}(\tau)f(\tau)d\tau = x_1 \ . \tag{6}$$

From here we have equality for determining a parameter μ:

$$\Phi(T)\int_0^T \Phi^{-1}(\tau)B(\tau)d\tau \cdot \mu = x_1 - \Phi(T)x_0 - \Phi(T)\int_0^T \Phi^{-1}(\tau)f(\tau)d\tau \ . \tag{7}$$

Suppose that the matrix $\Phi(T)\int_0^T \Phi^{-1}(\tau)B(\tau)d\tau$ is invertible, i.e. $\det\left[\Phi(T)\int_0^T \Phi^{-1}(\tau)B(\tau)d\tau\right] \neq 0$.

Then, from equation (7) we uniquely determine the parameter μ:

$$\mu = \left[\Phi(T)\int_0^T \Phi^{-1}(\tau)B(\tau)d\tau\right]^{-1}\{x_1 - \Phi(T)x_0\} - \left[\Phi(T)\int_0^T \Phi^{-1}(\tau)B(\tau)d\tau\right]^{-1}\Phi(T)\int_0^T \Phi^{-1}(\tau)f(\tau)d\tau. \tag{8}$$

Substituting the found parameter from (8) instead of μ into representation (6), we get

$$x(t) = \Phi(t)x_0 + \Phi(t)\int_0^t \Phi^{-1}(\tau)B(\tau)d\tau \cdot \left[\Phi(T)\int_0^T \Phi^{-1}(\tau)B(\tau)d\tau\right]^{-1}\{x_1 - \Phi(T)x_0\} -$$

$$-\Phi(t)\int_0^t \Phi^{-1}(\tau)B(\tau)d\tau \cdot \left[\Phi(T)\int_0^T \Phi^{-1}(\tau)B(\tau)d\tau\right]^{-1}\Phi(T)\int_0^T \Phi^{-1}(\tau)f(\tau)d\tau + \Phi(t)\int_0^t \Phi^{-1}(\tau)f(\tau)d\tau .$$

Therefore, we find the pair $(x^*(t), \mu^*)$ in the next form:

$$x^*(t) = \Phi(t)x_0 + \Phi(t)\int_0^t \Phi^{-1}(\tau)B(\tau)d\tau \cdot \left[\Phi(T)\int_0^T \Phi^{-1}(\tau)B(\tau)d\tau\right]^{-1}\{x_1 - \Phi(T)x_0\} -$$

$$-\Phi(t)\int_0^t \Phi^{-1}(\tau)B(\tau)d\tau \cdot \left[\Phi(T)\int_0^T \Phi^{-1}(\tau)B(\tau)d\tau\right]^{-1}\Phi(T)\int_0^T \Phi^{-1}(\tau)f(\tau)d\tau + \Phi(t)\int_0^t \Phi^{-1}(\tau)f(\tau)d\tau , \tag{9}$$

$$\mu^* = \left[\Phi(T)\int_0^T \Phi^{-1}(\tau)B(\tau)d\tau\right]^{-1}\{x_1 - \Phi(T)x_0\} - \left[\Phi(T)\int_0^T \Phi^{-1}(\tau)B(\tau)d\tau\right]^{-1}\Phi(T)\int_0^T \Phi^{-1}(\tau)f(\tau)d\tau, \quad (10)$$

This is the unique solution to problem (1)-(3).

The following assertion is true.

Theorem 1. *Let*

a) the $(n\times n)$ *matrices* $A(t)$, $B(t)$, *and* n *vector function* $f(t)$ *be continuous on* $[0,T]$, x_0, x_1 *be constant vectors;*

b) $\det\left[\Phi(T)\int_0^T \Phi^{-1}(\tau)B(\tau)d\tau\right]\neq 0$, i.e. the matrix $\Phi(T)\int_0^T \Phi^{-1}(\tau)B(\tau)d\tau$ be invertible, where $\Phi(t)$ *be the fundamental matrix of system* $\dfrac{dx}{dt} = A(t)x$ *and* $\Phi(0)=I$.

Then problem (1)-(3) has a unique solution $(x^*(t),\mu^*)$, *where the function* $x^*(t)$ and parameter μ^* *have the representations (9) and (10), respectively.*

It is well known that the construction of the fundamental matrix is rather difficult. Then, verification of condition b) becomes hard. Therefore, finding conditions in the terms of the initial data is of great interest.
The goal this work is finding coefficient condition for unique solvability to problem (1)—(3).
For this, we use the Dzhumabaev's parametrization method.
Using initial condition (2) we integrate system (1) by t:

$$x(t) = x_0 + \int_0^t A(\tau)x(\tau)d\tau + \int_0^t B(\tau)d\tau\cdot\mu + \int_0^t f(\tau)d\tau, \quad t\in[0,T]. \quad (11)$$

That is Volterra integral equation second kind with respect to function $x(t)$. Substituting right-hand side of (11) for $t=\tau$ instead of $x(\tau)$, and repeating this process m times, we obtain

$$x(t) = D_{0,m}(t)x_0 + D_m(t)\mu + G_m(t,x) + F_m(t), \qquad t\in[0,T], \quad (12)$$

where

$$D_{0,m}(t) = I + \int_0^t A(\tau)d\tau + \int_0^t A(\tau_1)\int_0^{\tau_1} A(\tau_2)d\tau_2 d\tau_1 + \ldots + \int_0^t A(\tau_1)\int_0^{\tau_1} A(\tau_2)\ldots\int_0^{\tau_{m-1}} A(\tau_m)d\tau_m\ldots d\tau_2 d\tau_1,$$

$$D_m(t) = \int_0^t B(\tau)d\tau + \int_0^t A(\tau_1)\int_0^{\tau_1} B(\tau_2)d\tau_2 d\tau_1 + \int_0^t A(\tau_1)\int_0^{\tau_1} A(\tau_2)\int_0^{\tau_2} B(\tau_3)d\tau_3 d\tau_2 d\tau_1 +$$

$$+\ldots+\int_0^t A(\tau_1)\int_0^{\tau_1} A(\tau_2)\ldots\int_0^{\tau_{m-2}} A(\tau_{m-1})\int_0^{\tau_{m-1}} B(\tau_m)d\tau_m d\tau_{m-1}\ldots d\tau_2 d\tau_1,$$

$$F_m(t) = \int_0^t f(\tau)d\tau + \int_0^t A(\tau_1)\int_0^{\tau_1} f(\tau_2)d\tau_2 d\tau_1 + \int_0^t A(\tau_1)\int_0^{\tau_1} A(\tau_2)\int_0^{\tau_2} f(\tau_3)d\tau_3 d\tau_2 d\tau_1 +$$

$$+\ldots+\int_0^t A(\tau_1)\int_0^{\tau_1} A(\tau_2)\ldots\int_0^{\tau_{m-2}} A(\tau_{m-1})\int_0^{\tau_{m-1}} f(\tau_m)d\tau_m d\tau_{m-1}\ldots d\tau_2 d\tau_1,$$

$$G_m(t,x) = \int_0^t A(\tau_1)\int_0^{\tau_1} A(\tau_2)\ldots\int_0^{\tau_{m-2}} A(\tau_{m-1})\int_0^{\tau_{m-1}} A(\tau_m)x(\tau_m)d\tau_m d\tau_{m-1}\ldots d\tau_2 d\tau_1, \quad m=1,2,\ldots .$$

We find the value of function $x(t)$ for $t=T$ and substitute it instead of $x(T)$ into relation (3). Then we have

$$D_{0,m}(T)x_0 + D_m(T)\mu + G_m(T,x) + F_m(T) = x_1. \quad (13)$$

From here we obtain the following system of algebraic equations with respect to μ:

$$D_m(T)\mu = x_1 - D_{0,m}(T)x_0 - G_m(T,x) - F_m(T), \qquad \mu \in R^n. \tag{14}$$

Let for some m ($m = 1,2,....$) the matrix $D_m(T)$ be invertible.

Then from system (14) we can define μ:

$$\mu = [D_m(T)]^{-1}\{x_1 - D_{0,m}(T)x_0\} - [D_m(T)]^{-1}G_m(T,x) - [D_m(T)]^{-1}F_m(T).$$

If the parameter μ is known, from Volterra integral equation (11) we get the function $x(t)$ for all $t \in [0,T]$. Then, we find the pair $(x(t),\mu)$ is the solution of problem (1)--(3).

If a function $x(t)$ is known, from the system of algebraic equations (14) we define the parameter. Then, we get the solution of problem (1)--(3).

Since, function $x(t)$ and parameter μ are unknown, we use an iterative process for finding solution of problem (1)--(3).

The sequential approximations of the pairs $(x^{(k)}(t),\mu^{(k)})$ are defined from the following algorithm.

Step 0. 1) Using initial condition (2) and solving the system of algebraic equations (14) for $x(t) = x_0$, we get an initial approximation $\mu^{(0)}$; 2) Solving the integral equation (11) for $\mu = \mu^{(0)}$, we find $x^{(0)}(t)$ for all $t \in [0,T]$.

Step 1. 1) Solving the system of algebraic equations (14) for $x(t) = x^{(0)}(t)$, we get a first approximation $\mu^{(1)}$. 2) Solving the integral equation (11) for $\mu = \mu^{(1)}$, we find $x^{(1)}(t)$ for all $t \in [0,T]$.

And so on.

Step k. 1) Solving the system of algebraic equations (14) for $x(t) = x^{(k-1)}(t)$, we get k th approximation $\mu^{(k)}$. 2) Solving the integral equation (11) for $\mu = \mu^{(k)}$, we find $x^{(k)}(t)$ for all $t \in [0,T]$.

$k = 1,2,...$.

The condition for realizability of the algorithm is invertibility of matrix $D_m(T)$ for some m ($m \in N$).

Now, it is important to find out the conditions of convergence of the proposed algorithm, which ensure uniform convergence sequence of pairs $(x^{(k)}(t),\mu^{(k)})$ to pair $(x^{*}(t),\mu^{*})$ is a solution of problem (1)--(3) as $k \to \infty$ for all $t \in [0,T]$.

We introduce the notations

$$\alpha = \max_{t\in[0,T]}\|A(t)\|, \quad \beta = \max_{t\in[0,T]}\|B(t)\|.$$

Theorem 2. *Let*

c) the $(n \times n)$ matrices $A(t)$, $B(t)$, and n vector function $f(t)$ be continuous on $[0,T]$, x_0, x_1 be constant vectors;

d) for some m ($m = 1,2,....$) the matrix $D_m(T)$ be invertible and $\|[D_m(T)]^{-1}\| \le \gamma_m(T)$, where $\gamma_m(T)$ is positive constant;

e) $q_m(T) = \gamma_m(T)\cdot\beta\cdot\alpha\cdot T\left[e^{\alpha T} - 1 - \alpha T - \frac{[\alpha T]^2}{2!} - ... - \frac{[\alpha T]^m}{m!}\right] < 1.$

Then the sequential approximations $x^{(k)}(t)$ and $\mu^{(k)}$, determined by the algorithm, converge uniformly to $x^{*}(t)$ and μ^{*}, respectively, as $k \to \infty$ for all $t \in [0,T]$. Moreover, the pair $(x^{*}(t),\mu^{*})$ is a unique solution to problem (1)--(3).

Proof of Theorem 2 is carried out according to the above algorithm.

For $m=1$ the matrix $D_1(T)$ has the form $D_1(T)=\int_0^T B(\tau)d\tau$.

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