МАТЕМАТИКА ЖӘНЕ ФИЗИКАЛЫҚ ПРОЦЕСТЕР МЕН МЕХАНИКАЛЫҚ ЖҮЙЕЛЕРДІ МАТЕМАТИКАЛЫҚ МОДЕЛЬДЕУ

МАТЕМАТИКА И МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ ФИЗИЧЕСКИХ ПРОЦЕССОВ И МЕХАНИЧЕСКИХ СИСТЕМ

MATHEMATICS AND MATHEMATICAL MODELING OF PHYSICAL PROCESSES AND MECHANICAL SYSTEMS

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MATHEMATICAL MODELING FOR OPTIMIZING GRAIN DRYING PRIOR TO MILLING

Abstract

This study presents a mathematical description of grain pre-drying formulated as a coupled system of ordinary differential equations that captures the essential heat and mass exchange between the grain and the drying agent. The model incorporates the principal operational and material parameters, including the initial moisture content of the grain and the effective heat and mass transfer coefficients. A parametric investigation is performed to quantify the influence of these factors on dehydration kinetics and on the time required to reach technologically acceptable moisture levels. The numerical predictions are evaluated against experimental measurements and demonstrate statistically acceptable agreement, supporting the adequacy of the proposed formulation for engineering analysis. The developed framework can be employed for preliminary estimation of drying duration, sensitivity assessment of process parameters, and as a foundation for subsequent extensions that account for spatial gradients of moisture and temperature in distributed drying models.

Keywords: grain drying, mathematical modeling, heat and mass transfer, parametric analysis, model adequacy.

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МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ ПРОЦЕССА СУШКИ ЗЕРНА ПЕРЕД ПОМОЛОМ

Аннотация

В статье представлено математическое моделирование процесса предварительной сушки зерна на основе системы связанных обыкновенных дифференциальных уравнений тепломассопереноса. Рассмотрены основные параметры процесса – начальная влажность зерна, коэффициенты тепло- и массообмена. Проведен параметрический анализ влияния этих факторов на кинетику обезвоживания. Численные результаты сопоставлены с экспериментальными данными и подтверждают адекватность модели. Предложенная модель может быть использована для анализа времени сушки и влияния ключевых параметров на динамику процесса, а также служит основой для дальнейшего развития моделей с пространственными градиентами влажности.

Ключевые слова: сушка зерна, математическое моделирование, тепломассоперенос, параметрический анализ, адекватность модели.

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ДӘНДІ ҰНТАҚТАУ АЛДЫНДА КЕПТІРУ ПРОЦЕСІН МАТЕМАТИКАЛЫҚ МОДЕЛЬДЕУ

Аңдатпа

Мақалада жылу және масса алмасу процестерін сипаттайтын өзара байланысқан қарапайым дифференциалдық теңдеулер жүйесіне негізделген дәнді алдын ала кептіру процесінің математикалық модельдеу процесі келтірілген. Процестің негізгі параметрлері ретінде дәннің бастапқы ылғалдылығы, жылу және масса алмасу коэффициенттері қарастырылған. Аталған факторлардың сусыздану кинетикасына әсерін бағалау мақсатында параметрлік талдау жүргізілді. Сандық нәтижелер эксперименттік деректермен салыстырылып, модельдің адекваттылығын растайды. Ұсынылған модель кептіру уақыты мен негізгі параметрлердің процесс динамикасына әсерін талдауға мүмкіндік береді, сондай-ақ ылғалдылықтың кеңістіктік градиенттерін ескеретін модельдерді одан әрі дамыту үшін негіз болып табылады.

Түйін сөздер: дәнді кептіру, математикалық модельдеу, жылу және масса алмасу, параметрлік талдау, модельдің адекваттылығы.

Introduction

Drying of grain with elevated initial moisture content remains one of the most energy-demanding operations in post-harvest processing and flour-milling technology [1-4]. From a theoretical standpoint, the process is commonly described through coupled heat and mass transfer relations together with kinetic equations governing the evolution of grain moisture content over time [5-8]. Comprehensive overviews of contemporary drying technologies, their physical principles, practical implementation, and modeling approaches are provided in [9], while targeted strategies aimed at lowering energy consumption during grain dehydration are discussed in [10].

In recent years, the use of mathematical modeling and numerical simulation has intensified as a means of improving drying performance for grain and other agricultural products. For example, Zhang et al. [11] formulated a corn-drying model derived from energy and mass conservation laws, implemented it in MATLAB, and validated the predicted dehydration curves against experimental observations. Wang et al. [12] investigated nonlinear models of wheat drying in a fluidized-bed configuration using finite-element discretization, reporting accurate approximation of moisture-loss trajectories and potential reductions in energy demand relative to conventional regimes. The role of equipment geometry and airflow organization has been further explored by Chen et al. [13] through CFD simulations, demonstrating their influence on the uniformity of temperature and moisture fields within the drying chamber. Rahman et al. [14] showed that adaptive control schemes supported by artificial neural-network models can provide real-time moisture prediction, thereby improving product quality and limiting avoidable thermal losses.

Unlike systems such as those considered in [15], where interphase mass transfer is governed predominantly by dynamic factors, grain drying in industrial practice is often constrained by physicochemical mechanisms of heat and mass transfer, including evaporation at the surface and internal moisture migration. The literature reviewed above therefore reinforces the practical value of embedding mathematical models into the monitoring and control loop of drying units used at flour-milling enterprises. At the same time, despite clear progress, the engineering adoption of heat- and mass-transfer based models and robust numerical solution techniques remains limited, and their capabilities are not yet fully exploited in routine process design and operation.

Against this background, the present study proposes a mathematical description of preliminary grain drying formulated as a system of coupled ordinary differential equations representing heat and mass exchange between the grain and the drying agent. The scientific contribution of the work lies in providing a rigorous justification of the associated Cauchy problem: under physically meaningful constraints on the parameters, the existence and uniqueness of the solution are established, and the model behavior is examined through a parametric study that quantifies how the initial grain moisture content and the heat and mass transfer coefficients affect dehydration kinetics. From an applied perspective, the model (i) enables systematic evaluation of the sensitivity of drying dynamics to key operating parameters, (ii) supports preliminary estimates of drying duration and qualitative analysis of solution trajectories, and (iii) offers a tractable foundation for subsequent extensions that incorporate spatial moisture gradients and more realistic initial and boundary conditions tailored to specific dryer configurations.

Research methodology. To characterize the temporal evolution of the main drying-process variables, a lumped-parameter mathematical model was formulated in the form of coupled ordinary differential equations (ODEs). The model follows classical heat and mass transfer considerations while preserving a level of simplicity suitable for preliminary engineering analysis. The following assumptions were adopted:

Spatial averaging of moisture content. The grain moisture content is treated primarily as a function of time. Although moisture may vary within the grain layer and along spatial coordinates, these gradients are neglected in the present first-order description.

Constant transfer parameters. Heat and mass transfer coefficients are assumed not to depend on local physicochemical conditions and are taken as constant during the considered drying interval.

Uniform supply of the drying agent. Mixing and segregation effects in the grain bed are not modeled; the hot air is assumed to be supplied uniformly to the grain volume.

Assumptions (1)-(3) reduce model fidelity relative to distributed (PDE-based) formulations; however, they allow the moisture and temperature dynamics to be expressed as a tractable ODE system. The obtained numerical solutions were subsequently compared with available experimental data to verify that the model provides a reasonable approximation under typical industrial conditions.

The principal state variables and parameters are defined as follows:

X(t) – grain moisture content, kg moisture/kg dry matter;

Tg(t) – drying-air temperature, °C;

Tz(t) – grain temperature, °C;

h – convective heat transfer coefficient between air and grain, $W/(m^2 \cdot K)$;

A – effective heat/mass transfer area, m^2;

ρg – density of the drying agent (air), kg/m^3;

cg – specific heat of air, J/(kg·K);

cz – specific heat of grain, J/(kg·K);

L – latent heat of moisture evaporation, J/kg;

km – mass transfer coefficient, $kg/(m^2 \cdot s)$;

Wg (t)– absolute humidity of air, kg water/kg dry air;

Wge – equilibrium air humidity under the specified conditions, kg water/kg dry air.

mz – mass of processed grain, kg;

mg - mass of drying air participating in heat exchange (lumped equivalent), kg.

The drying process was described by the following system [6–7]:

1. Moisture balance for grain

$$\frac{dX}{dt} = -k_m \mathrm{A}(\mathrm{W_g} - \mathrm{W_{ge}}), \tag{1}$$

where $(\mathrm{W_g} - \mathrm{W_{ge}})$- represents the driving force for moisture removal in the adopted formulation.

2. Heat balance for grain,

$$\frac{dTz}{dt} = \frac{A}{m_z c_z}(T_g - T_z) - \frac{L}{c_z}\frac{dX}{dt}. \tag{2}$$

3. Heat balance for air

$$\frac{dT_g}{dt} = -\frac{A}{m_g c_g}(T_g - T_z). \tag{3}$$

4. Humidity balance for air

$$\frac{dW_g}{dt} = \frac{k_m A}{\rho_g}(X - W_g). \tag{4}$$

Equations (1)-(4) jointly couple moisture loss, phase-change effects, and heat exchange between the grain and the drying agent. Their combined analysis enables tracking the interdependence of grain moisture content, grain temperature, air temperature, and air humidity throughout the drying interval.

The Cauchy problem is completed by the following initial conditions:

$$X(0) = X_0, \tag{5}$$
$$T_z(0) = T_{z0}, \tag{6}$$
$$T_g(0) = T_{g0}, \tag{7}$$
$$W_g(0) = W_{g0}, \tag{8}$$

The coefficients h, km and A depend on the drying regime, airflow velocity, and grain size. If required, the framework can be extended by introducing additional relations for airflow dynamics; in the present study, averaged values typical for industrial grain drying were used:

h=30 W/(m²K);
A=5 m²;
ρg =1.2 kg/m³;
cg=1005 J/(kg K);
cz=2000 J/(kgK);
L=2.25·106 J/kg;
km=0.02 kg/(m²);
Wg = 0.015 kg of water/kg of dry air;
Wge = 0.012 kg of water/kg of dry air.

The ODE system (1)-(8) was solved numerically using the classical fourth-order Runge-Kutta method with adaptive step-size control to ensure stable integration and adequate accuracy across regimes with different initial moisture contents and transfer coefficients.

Let the system (1)-(4) be written in vector form:

$$y'(t) = F(t,y),\ y(0) = y0, \tag{9}$$

where $y = (w, Tg, Ta, Y)^T$, and the model parameters (h, k, A, ρa, ca, cg, L, …) are positive finite constants. If F is continuous in t and locally Lipschitz in y on a domain D ⊂ R4 containing y0, then by the Picard-Lindelof theorem, there exists a unique local solution y(t) on [0, t*). If, in addition, F satisfies a linear growth bound

$$|F(t,y)| \le C(1+|y|) \tag{10}$$

For all $y \in D$ and some C > 0, the solution extends to any finite interval [0,T]. Moreover, for physically admissible initial data $X_0 \ge 0$ and $W_{g0} \ge 0$, the corresponding components remain non-negative throughout the interval of existence, consistent with the physical meaning of moisture-related state variables.

Research results. The numerical integration of the coupled heat and mass-transfer model yields time-dependent trajectories for grain moisture content and related state variables. The predicted moisture decay curves are summarized in Figure 1, where three initial moisture levels (X0=35%, 32%, and 27%) are compared. In all cases, the moisture content decreases monotonically and approaches the target technological range. However, the time required to reach the commonly specified residual moisture (approximately 8%) varies substantially with the initial condition.

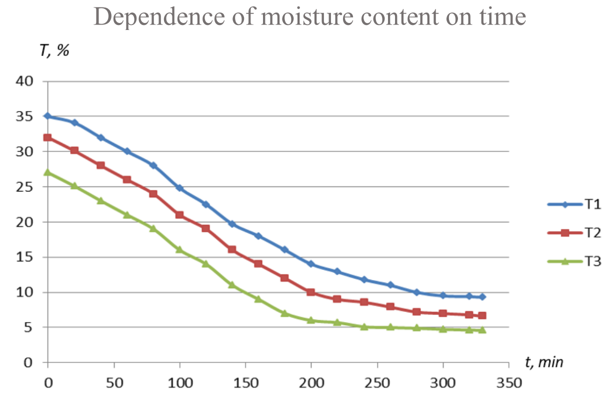

Figure 1. Dependence of moisture content on time for initial moisture content of 35%, 32%, and 27%

A sensitivity assessment was performed to clarify how the principal parameters influence dehydration kinetics. Figure 2 illustrates the effect of initial grain moisture on the drying curve, confirming that higher X0 leads to a longer dehydration period under the same external regime.

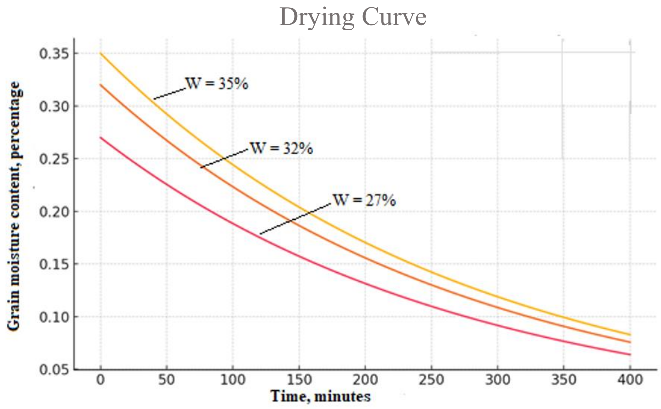

Figure 2. Effect of initial moisture content on drying kinetics

The influence of convective heat exchange is shown in Figure 3: increasing the heat transfer coefficient intensifies heat supply to the grain, accelerates phase change, and shortens the time needed to reduce moisture.

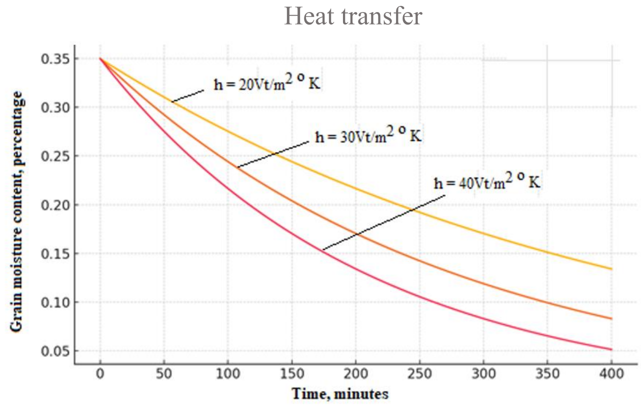

Figure 3. Effect of heat transfer coefficient on drying kinetics

Similarly, Figure 4 demonstrates that larger mass transfer coefficients increase the moisture-removal rate by strengthening the vapor moisture exchange between the grain surface and the drying agent.

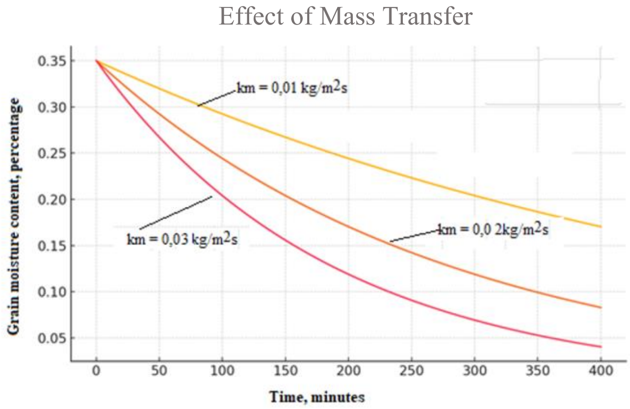

Figure 4. Effect of mass transfer coefficient on drying kinetics

To evaluate the agreement between numerical predictions and experimental observations, the adequacy of the model was tested using a paired Student's t-test in accordance with standard procedures for statistical validation of mathematical models [16,17]. Let X1(ti) denote the model-predicted moisture content and X2(ti) the experimentally measured value at the same sampling time ti. The paired differences are computed as

$$d_i = X_1\,(t_i) - X_2(t_i). \tag{11}$$

The main difference is:

$$\bar{d} = \frac{1}{n}\sum_{i=1}^{n} d_i\,\alpha. \tag{12}$$

and the sample standard deviation of differences is

$$s_d = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(d_i - \bar{d}\right)^2}. \tag{13}$$

The test statistic is then

$$t = \frac{\bar{d}}{s_d/\sqrt{n}} \tag{14}$$

For the considered dataset (n = 8), the degrees of freedom are f = n−1=7. Under the conditions of this study, the calculated statistic is t =2.00, which corresponds to a two-sided probability value of approximately $p \approx 0.085$. Since $p > 0.05$ at the significance level α=0.05, the null hypothesis of zero mean difference cannot be rejected. Therefore, the discrepancies between the modeled and experimental moisture contents are not statistically significant at the 5% level, and the model may be regarded as adequate for approximate engineering estimates within the studied regime. The paired-samples test was performed for the case X0=35%. The experimental and simulated values, as well as the difference calculations, are provided in Tables 1-2.

For this dataset, $\bar{d} = 0.017004, n = 8$, and the computed statistic is t =2.002 with p = 0.085. Consequently, at α = 0.05, the model experiment differences are statistically insignificant, supporting the adequacy of the proposed model for preliminary drying-time calculations and regime comparison.

Table 1. Initial data

№	Time, min	Model moisture (X1)	Experimental moisture (X2)
1	0	0.350	0.341
2	50	0.290	0.301
3	100	0.240	0.231
4	150	0.190	0.181
5	200	0.140	0.145
6	250	0.110	0.072
7	300	0.080	0.046
8	350	0.080	0.069

Table 2. Differences between paired values

№	Difference ($d_i = X1 - X_2$)	$(d_i - \bar{y})^2$
1	0.0094	0.000007
2	-0.0109	0.000524
3	0.0093	0.000008
4	0.0093	0.000007
5	-0.0048	0.000285
6	0.0383	0.000688
7	0.0345	0.000505
8	0.0112	0.000001

Discussion

The performed simulations make it possible to distinguish the parameters that most strongly govern grain dehydration under the adopted drying regime. In the proposed lumped formulation, the drying duration is controlled primarily by the initial grain moisture content and by the intensities of heat and mass exchange between the grain and the drying agent, represented through the corresponding transfer coefficients. The parametric trends obtained from the numerical solution are physically consistent: higher initial moisture content increases the amount of water to be removed and therefore prolongs the process, whereas increases in the heat and mass transfer coefficients accelerate moisture removal by enhancing energy delivery for evaporation and strengthening the moisture-vapor exchange with the air stream.

The model predicts that the conventional target moisture range for milling preparation (approximately 8-10%) is reached at markedly different times depending on the initial condition. For the considered parameter set, the required moisture level is achieved after about 300 min for X0=35%, 250 min for X0=32%, and 170 min for X0=27%. These estimates provide a practical, first-order characterization of dehydration kinetics and offer a basis for comparison with experimental drying curves and for rapid evaluation of alternative operating regimes.

From an engineering standpoint, the developed ODE-based model is therefore suitable for preliminary calculations of drying time and for sensitivity analysis aimed at identifying the most influential process parameters. At the same time, the simplifying assumptions imply that spatial non-uniformities in temperature and moisture are not represented. A natural direction for further development is the transition to distributed formulations based on partial differential equations that account for moisture gradients within the grain layer (and, if necessary, within the kernel), as well as more detailed initial and boundary conditions reflecting specific dryer geometry, airflow patterns, and nonstationary operating modes.

Conclusion

Grain conditioning prior to milling remains an energy-intensive stage of post-harvest processing, and its rational design requires an explicit understanding of the coupled heat and mass transfer mechanisms that govern moisture removal. In this work, a systems-based framework was developed in which the drying process is represented by a set of coupled ordinary differential equations describing the exchange of heat and moisture between the grain and the drying agent. The combination of model-based simulation and statistical adequacy assessment provides a practical tool for evaluating how operating and material parameters affect dehydration dynamics.

The numerical experiments confirm three key outcomes. First, the time required to reach the target technological moisture level exhibits pronounced sensitivity to the initial moisture content of the grain. Second, increases in the heat and mass transfer coefficients systematically intensify dehydration and shorten the drying period, reflecting enhanced energy delivery for evaporation and stronger moisture exchange with the air stream. Third, the formulation is readily adaptable to specific industrial conditions through parameter identification and by specifying appropriate initial states, which supports its use for preliminary engineering estimates.

A three-factor parametric analysis (initial moisture content, heat transfer coefficient, and mass transfer coefficient) demonstrates the dominant role of these parameters in shaping the drying kinetics; the resulting curves provide a clear visualization of the solution behavior and its sensitivity to regime changes. Comparison of simulated and experimental moisture data using a paired Student’s t-test indicates that the observed discrepancies are not statistically significant at the 5% significance level ($\alpha=0.05$, $p>0.05$), which supports the adequacy of the model for analyzing drying dynamics within the investigated range. The results also emphasize the inherently nonlinear nature of grain dehydration, where the evolution of moisture removal depends on both the initial moisture content and the thermal conditions of the process.

Further development of the presented approach is naturally associated with moving from a lumped description to distributed models formulated in two- and three-dimensional domains. Such an extension is motivated by the presence of spatial gradients of temperature and moisture in real dryers and by the influence of chamber geometry and airflow organization on local transfer rates. As reported in prior studies, multidimensional heat and mass transfer models provide a more realistic representation of moisture and temperature fields and improve the predictive accuracy of drying regimes [18-20]. Implementing these distributed formulations would therefore broaden the applicability of the proposed modeling framework for the analysis, design, and optimization of industrial grain drying systems.

Список использованных источников

[1] Яруллина Г.Р. Управление энергосбережением на промышленном предприятии. – Казань: КГФЭИ, 2003. -111 с.

[2] M. K. Bezrodny, V. V. Vovk. Energy efficiency of heat pump continuous flow grain dryers. Energy and Automation. – 2025. – No.3. https://doi.org/10.31548/energiya3(79).2025.125.

[3] Гончаренко В. М., Попов В. Ю. Методы оптимальных решений в экономике и финансах. – М.: КНОРУС, 2014. – 400 с.

[4] Al Momani D., Al Turk Y., Abuashour M.I., Khalid H.M., et al. Energy saving potential analysis applying factory scale energy audit – A case study of food production. Heliyon, 2023. Vol. 9, No.3. https://doi.org/10.1016/j.heliyon.2023.e14216.

[5] Уринбоев А.А., Исмаилов Б.Р. О проблемах разработки АСУ подготовки зерна на мукомольных комбинатах // Тр.межд.научн.-практ.конференции «Ауэзовские чтения - 22: Академик К. Сатпаев основоположник казахстанской науки». Шымкент: ЮКУ им. М.Ауэзова. – 2024. – Т. 5. – С. 209-213.

[6] Solaja O.A., Abiodun J.A., Abioro M., & Ekpudu J. E. Application of linear programming techniques in production planning. International Journal of Applied Operational Research. Computers and Industrial Engineering, 2019. Vol. 9, No.3, pp.11-19.

[7] Акулич И.Л. Математическое программирование в примерах и задачах. – СПб.: Лань. 2011. – 352 с. http://e.lanbook.com/view/book/2027/.

[8] Аттетков А. В., Галкин С. В., Зарубин В. С. Методы оптимизации. - М.: МГТУ им. Н. Э. Баумана, 2001. - 439 с.

[9] Атаназевич В.И. Сушка зерна. – М.: ДеЛи принт.2007. – 480 с.

[10] Малин Н.И. Энергосберегающая сушка зерна. – М.: Колос-С, 2004. – 240 с.

[11] S. Wei, Z. Wang, F. Wang, W. Xie, P. Chen, D. Yang. Simulation and experimental studies of heat and mass transfer in corn kernel during hot air drying. Food and Bioproducts Processing. 2019. Vol. 117, pp. 360-372. https://doi.org/10.1016/j.fbp.2019.08.006.

[12] Li X., Yang K., Wang Y., & Du X. Simulation study on coupled heat and moisture transfer in grain drying process based on discrete element and finite element method. Drying Technology. 2023. Vol. 41, No. 12, pp. 2027-2041. https://doi.org/10.1080/07373937.2023.2213767.

[13] Azmir J., Hou Q., & Yu A. CFD-DEM simulation of drying of food grains with particle shrinkage. Powder Technology. 2019. Vol. 343, pp. 792–802. https://doi.org/10.1016/J.POWTEC.2018.11.097.

[14] Liu X., Chen X., & Wu W. A neural network for predicting moisture content of grain drying process using genetic algorithm. Food Control. 2007. Vol. 18, No. 8, pp. 928-933. https://doi.org/10.1016/j.foodcont.2006.05.010.

[15] B. Ismailov, Kh. Ismailov, A.Shambilova, Zh. Iztaev, G. Tileuov. Simulation of Hydrodynamics and Mass Transfer in Separated Flows Past Packings in Technological Apparatuses. Chemical Engineering Transactions. 2023. Vol. 100, pp.349-354. https://doi.org/10.3303/CET23100059.

[16] Montgomery D. C., Peck E. A., & Vining G. G. Introduction to linear regression analysis. Wiley. 2012.

[17] Draper N. R., & Smith H. Applied regression analysis. Wiley. 1998. https://doi.org/10.1002/9781118625590.

[18] Datta A. K. Porous media approaches to studying simultaneous heat and mass transfer in food processes. I: Problem formulations. Journal of Food Engineering. 2007. Vol. 80, No. 1, pp. 80-95. https://doi.org/10.1016/j.jfoodeng.2006.05.013.

[19] Białobrzewski I., Zielińska M., Mujumdar A. S., & Markowski M. Heat and mass transfer during drying of a bed of shrinking particles – Simulation for carrot cubes dried in a spout-fluidized-bed drier. International Journal of Heat and Mass Transfer. 2008. Vol. 51, No. 19-20, pp. 4704-4716. https://doi.org/10.1016/j.ijheatmasstransfer.2008.02.031.

[20] Ranjan R., Irudayaraj J., Jun S. A three-dimensional control volume approach to modeling heat and mass transfer in foods materials. Transactions of the American Society of Agricultural Engineers. 2001. Vol. 44, No. 6, pp. 1975-1982. https://doi.org/10.13031/2013.6996.

References

[1] Yarullina G.R. (2003) Upravlenie energosberezheniem na promyshlennom predprijatii [Energy saving management at an industrial enterprise]. Kazan: Kazan State Institute of Finance and Economics. 111 p. (in Russian)

[2] M. K. Bezrodny, V. V. Vovk. Energy efficiency of heat pump continuous flow grain dryers. Energy and Automation. – 2025. – No. 3. https://doi.org/10.31548/energiya3(79).2025.125.

[3] Goncharenko V.M., Popov V.Yu. (2014) Metody optimal'nyh reshenij v jekonomike i finansah [Methods of optimal decision-making in economics and finance]. Moscow: KNORUS. 400 p. (in Russian)

[4] Al Momani D., Al Turk Y., Abuashour M.I., Khalid H.M., et al. Energy saving potential analysis applying factory scale energy audit – A case study of food production. Heliyon. – 2023. – Vol. 9, No. 3. https://doi.org/10.1016/j.heliyon.2023.e14216.

[5] Urinboev A.A., Ismailov B.R. O problemah razrabotki ASU podgotovki zerna na mukomol'nyh kombinatakh [On the problems of developing automated control systems for grain preparation at flour mills]. Tr. mezhd. nauchn.-prakt. konf. «Aujezovskie chtenija – 22». Shymkent: M. Auezov South Kazakhstan University. – 2024. – Vol. 5. – Pp. 209-213. (in Russian)

[6] Solaja O.A., Abiodun J.A., Abioro M., Ekpudu J.E. Application of linear programming techniques in production planning. International Journal of Applied Operational Research. Computers and Industrial Engineering. – 2019. – Vol. 9, No. 3. – Pp. 11-19.

[7] Akulich I.L. (2011) Matematicheskoe programmirovanie v primerah i zadachah [Mathematical programming: examples and problems]. Saint Petersburg: Lan Publishing House. 352 p. http://e.lanbook.com/view/book/2027/. (in Russian)

[8] Attetkov A.V., Galkin S.V., Zarubin V.S. (2001) Metody optimizacii [Optimization methods]. Moscow: Bauman Moscow State Technical University. 439 p. (in Russian)

[9] Atanazevich V.I. (2007) Sushka zerna [Grain drying]. Moscow: DeLi Print. 480 p. (in Russian)

[10] Malin N.I. (2004) Energosberegajushhaja sushka zerna [Energy-saving grain drying]. Moscow: Kolos-S. 240 p. (in Russian)

[11] S. Wei, Z. Wang, F. Wang, W. Xie, P. Chen, D. Yang. Simulation and experimental studies of heat and mass transfer in corn kernel during hot air drying. Food and Bioproducts Processing. – 2019. – Vol. 117. – Pp. 360-372. https://doi.org/10.1016/j.fbp.2019.08.006.

[12] Li X., Yang K., Wang Y., Du X. Simulation study on coupled heat and moisture transfer in grain drying process based on discrete element and finite element method. Drying Technology. – 2023. – Vol. 41, No. 12. – Pp. 2027-2041. https://doi.org/10.1080/07373937.2023.2213767.

[13] Azmir J., Hou Q., Yu A. CFD–DEM simulation of drying of food grains with particle shrinkage. Powder Technology. – 2019. – Vol. 343. – Pp. 792–802. https://doi.org/10.1016/j.powtec.2018.11.097.

[14] Liu X., Chen X., Wu W. A neural network for predicting moisture content of grain drying process using genetic algorithm. Food Control. – 2007. – Vol. 18, No. 8. – Pp. 928-933. https://doi.org/10.1016/j.foodcont.2006.05.010.

[15] Ismailov B., Ismailov Kh., Shambilova A., Iztaev Zh., Tileuov G. Simulation of hydrodynamics and mass transfer in separated flows past packings in technological apparatuses. Chemical Engineering Transactions. – 2023. – Vol. 100. – Pp. 349-354. https://doi.org/10.3303/CET23100059.

[16] Montgomery D.C., Peck E.A., Vining G.G. (2012) Introduction to linear regression analysis. Wiley.

[17] Draper N.R., Smith H. (1998) Applied regression analysis. Wiley. https://doi.org/10.1002/9781118625590.

[18] Datta A.K. Porous media approaches to studying simultaneous heat and mass transfer in food processes. I: Problem formulations. Journal of Food Engineering. – 2007. – Vol. 80, No. 1. – Pp. 80-95. https://doi.org/10.1016/j.jfoodeng.2006.05.013.

[19] Białobrzewski I., Zielińska M., Mujumdar A.S., Markowski M. Heat and mass transfer during drying of a bed of shrinking particles – Simulation for carrot cubes dried in a spout-fluidized-bed drier. International Journal of Heat and Mass Transfer. – 2008. – Vol. 51, No. 19–20. – Pp. 4704-4716. https://doi.org/10.1016/j.ijheatmasstransfer.2008.02.031.

[20] Ranjan R., Irudayaraj J., Jun S. A three-dimensional control volume approach to modeling heat and mass transfer in food materials. Transactions of the American Society of Agricultural Engineers. – 2001. – Vol. 44, No. 6. – Pp. 1975-1982. https://doi.org/10.13031/2013.6996.