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SYNTHESIS OF SELF-ORGANIZING CONTROL SYSTEMS IN THE CLASS OF TWO-PARAMETER STRUCTURALLY STABLE MAPPINGS

Abstract

This paper presents a methodology for the synthesis of a self-organizing control system for a single-input single-output object within the class of two-parameter structurally stable mappings. The proposed approach is based on a Lyapunov framework using a gradient-type method applied to Lyapunov vector functions. Conditions defining aperiodic robust stability of stationary states are derived for both the control system and its corresponding model with prescribed transient characteristics. These conditions are formulated as parameter inequalities that define admissible regions ensuring non-oscillatory convergence of system trajectories. The obtained stability conditions are used to compute the controller coefficients, establishing a relationship between system parameters and admissible stability regions. Simulation results show that the state trajectories converge to the equilibrium point, while the control signal remains bounded. The transient response is well damped and does not exhibit oscillatory behavior. The results demonstrate that the proposed approach can be applied to the design of control systems operating under parametric uncertainty.

Keywords: self-organizing control systems, aperiodic robust stability, gradient-rate method of Lyapunov vector functions.

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ЕКІ ПАРАМЕТРЛІ ҚҰРЫЛЫМДЫҚ ТҰРАҚТЫ БЕЙНЕЛЕУЛЕР КЛАСЫНДАҒЫ ӨЗДІГІНЕН ҰЙЫМДАСТЫРЫЛАТЫН БАСҚАРУ ЖҮЙЕЛЕРІНІҢ СИНТЕЗІ

Аңдатпа

Бұл мақалада екі параметрлі құрылымдық тұрақты бейнелеулер класындағы бір кірісті, бір шығысты қондырғы үшін өзін-өзі ұйымдастыратын басқару жүйесін синтездеу әдістемесі ұсынылған. Ұсынылған тәсіл Ляпунов градиентті-жылдамдықтық вектор-функция әдісі арқылы негізделген. Стационарлық күйлердің апериодтық робастты тұрақтылығының шарттары басқару жүйесі үшін де, берілген өтпелі сипаттамалары бар сәйкес модель үшін де алынды. Бұл шарттар жүйенің траекторияларының тербелмелі емес конвергенциясын қамтамасыз етілетін рұқсат етілген аймақтарды анықтайтын параметрлік теңсіздіктер түрінде тұжырымдалған. Алынған тұрақтылық шарттары жүйе параметрлері мен рұқсат етілген тұрақтылық аймақтары арасындағы байланысты орната отырып, контроллер коэффициенттерін есептеу үшін қолданылады. Модельдеу нәтижелері күй траекторияларының тепе-теңдік нүктесіне жақындайтынын, ал басқару әрекеті шектеулі болып қалатынын көрсетеді. Өтпелі процесс сөніп қалады және тербелмелі сипат көрсетпейді. Алынған нәтижелер ұсынылған тәсілді параметрлік белгісіздік жағдайында жұмыс істейтін басқару жүйелерін жобалауда тиімді қолдануға болатынын көрсетеді.

Түйін сөздер: Өзін-өзі ұйымдастырылатын басқару жүйелері, апериодты робастты тұрақтылық, Ляпуновтың градиентті-жылдамдықтық вектор-функция әдісі.

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СИНТЕЗ САМООРГАНИЗУЮЩИХСЯ СИСТЕМ УПРАВЛЕНИЯ В КЛАССЕ ДВУХПАРАМЕТРИЧЕСКИХ СТРУКТУРНО УСТОЙЧИВЫХ ОТОБРАЖЕНИЙ

Аннотация

В данной статье представлена методология синтеза самоорганизующейся системы управления для объекта с одним входом и одним выходом в классе двухпараметрических структурно устойчивых отображений. Предложенный подход основан на градиентно-скоростном методе вектор функций Ляпунова. Выведены условия апериодической робастной устойчивости стационарных состояний как для системы управления, так и для соответствующей модели с заданными переходными характеристиками. Эти условия сформулированы в виде параметрических неравенств, определяющих допустимые области, в пределах которых обеспечивается неколебательная сходимость траекторий системы. Полученные условия устойчивости используются для вычисления коэффициентов регулятора, устанавливая связь между параметрами системы и допустимыми областями устойчивости. Результаты моделирования показывают, что траектории состояний сходятся к точке равновесия, при этом управляющее воздействие остается ограниченным. Переходный процесс является затухающим и не имеет колебательного характера. Полученные результаты свидетельствуют о том, что предложенный подход может быть эффективно использован при проектировании систем управления, функционирующих в условиях параметрической неопределенности.

Ключевые слова: самоорганизующиеся системы управления, апериодическая робастная устойчивость, градиентно-скоростной метод вектор функций Ляпунова.

Introduction

Over the past decades, nonlinear dynamical systems have been shown to exhibit a wide range of behaviors, including instability, bifurcations, and chaotic motion. One of the key developments in this field is the concept of deterministic chaos and the emergence of so-called strange attractors, which significantly influence the dynamics of complex systems [1-4].

In engineering applications, such phenomena may manifest as uncontrolled or runaway processes that can lead to system failures. In other domains, including economic, environmental, and biological systems, deterministic chaos is often associated with oscillatory and fluctuating behavior that may trigger critical transitions or crises [3,4].

The presence of chaotic dynamics complicates the design of control systems, since nonlinear systems are inherently capable of generating unpredictable trajectories. As a result, considerable research efforts have been devoted to the suppression or mitigation of chaotic behavior and to the development of methods that ensure stable system operation [1,4].

In practice, control systems typically operate under uncertainty, which may arise from parameter variations, external disturbances, or modeling inaccuracies. Under such conditions, even linear systems may lead to instability or complex dynamic behavior when combined with nonlinear control laws or uncertainties, especially when robust stability of the equilibrium states is not guaranteed [2,5,6]. Violation of robust stability conditions may lead to instability and, in nonlinear systems, potentially to chaotic behavior.

To address these challenges, self-organizing control systems have been proposed as an effective approach for maintaining stability under uncertainty [7,8]. These systems are often constructed using two-parameter structurally stable mappings, which allow the system behavior to vary depending on the current state and parameters [10,11].

A characteristic feature of self-organizing control systems is the presence of multiple stationary states. These states may differ in their stability properties, and transitions between them can occur through bifurcation mechanisms when system parameters change [7,8,12]. Such transitions represent the process of self-organization, in which the system may transition between regimes with different stability properties.

In this context, ensuring robust stability of the stationary states becomes a key requirement for reliable system operation. The analysis of these properties provides a basis for the synthesis of control laws capable of preventing instability and suppressing undesirable dynamic regimes.

The synthesis problem of self-organizing control systems within the class of two-parameter structurally stable mappings can be decomposed into the following tasks:

- investigation and determination of the robust stability domain of the steady states of the self-organizing control system;
- selection, analysis, and determination of the robust stability domain of the steady states of the self-organizing model with specified parameters and desired transient responses;
- computation of the controller coefficients on the basis of the aperiodic robust stability conditions of both the model and the self-organizing control system.

The analysis of self-organizing control systems and models with prescribed transient characteristics is performed using the gradient-velocity method based on Lyapunov vector functions [7,8]. This approach allows the determination of parameter regions ensuring aperiodic robust stability for both the control system and its corresponding model with desired transient performance.

Consequently, self-organizing control systems and models belong to the class of aperiodically stable systems. In this class, the norm of the solutions of the state equations decreases monotonically. This corresponds to an aperiodic transient response that satisfies the required performance specifications. A self-organizing control system ensures the specified performance characteristics. Within these regions, the system demonstrates non-oscillatory convergence to the equilibrium state.

The applied method is based on a geometric interpretation of stability analysis associated with Lyapunov's second method [13-15]. It relies on representing system behavior in terms of Lyapunov vector functions, which allows the stability properties to be examined in the state space. In this framework, the system dynamics can be interpreted as gradient-type dynamics, conceptually related to ideas from catastrophe theory [11,12].

$$\left(\frac{dx_i}{dt}\right)_{x_j} = -\frac{\partial V_i(x)}{\partial x_j}, i = 1,...,n; j = 1,...,n.$$

In this formulation, $x(t) \in R^n$ - denotes the state vector of the dynamic system, is a Lyapunov vector function whose components are associated with the state variables.

Using the representation of gradient dynamical systems, the partial derivatives of the Lyapunov vector function with respect to the state variables $\left(\frac{\partial V_i(x)}{\partial x_j}, i = 1,...,n; j = 1,...,n\right)$ can be used to characterize the system behavior. In addition, the components of the velocity vector in the state space are obtained directly from the system equations in the form [16] $(x_1,...,x_n);\left(\frac{dx_i}{dt}\right)_{x_j}, i = 1,...,n; j = 1,...,n.$

The novelty of this study is to establish a unified framework for the synthesis and stability analysis of self-organizing control systems within the class of two-parameter structurally stable mappings. Unlike existing approaches, the proposed method makes it possible to explicitly determine admissible parameter regions ensuring aperiodic robust stability of both the control system and the corresponding model. The obtained conditions are expressed in terms of Lyapunov vector functions and provide a constructive basis for controller design. Furthermore, the developed method ensures non-oscillatory behavior within the derived parameter region and improves the robustness of the system with respect to parameter uncertainties.

Methodology

The paper examines how stability properties can be evaluated in self-organizing control systems that are based on structurally stable mappings.

Consider a linear dynamic system with a single input and a single output described by

$$\begin{aligned} \dot{x} &= Ax(t) + Bu(t) \\ y &= Cx \end{aligned} \tag{1}$$

Where $x(t) \in R^n$ is the state vector of the system. The matrices $A \in R^{n \times n}$, $B \in R^m$ and $C \in R^l$ define the system dynamics and input structure.

$$A = \begin{Vmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \\ a_n & a_{n-1} & a_{n-2} & \dots & a_1 \end{Vmatrix}, B = \begin{Vmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ b_n \end{Vmatrix}, X = \begin{Vmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Vmatrix}$$

The control input is constructed using a nonlinear mapping based on two-parameter structurally stable functions. In this case, the control law is expressed in the form [10,11].

$$u_i(t) = -x_i^4 - k_i^1 x_i^2 + k_i x_i, i = 1,...,n \tag{2}$$

Substituting the control law (2) into system (1), the closed-loop system can be expressed in component form as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \vdots \\ \dot{x}_{n-1} = x_n \\ \dot{x}_n = b_n \left[-x_1^4 - k_1^1 x_1^2 - k_1^1 x_1^2 + \left(k_1 + \dfrac{a_n}{b_n} \right) x_1 \right] + b_n \left[-x_2^4 - k_2^1 x_2^2 + \left(k_2 + \dfrac{a_{n-1}}{b_n} \right) x_2 \right] + \\ +,...,+b_n \left[-x_n^4 - k_n^1 x_n^2 + \left(k_n + \dfrac{a_1}{b_n} \right) x_n \right], \end{cases} \tag{3}$$

while the last equation includes nonlinear terms determined by the control input.

The equilibrium points $x_{is}, i = 1,...,n$ of the resulting system (3) are obtained by setting the time derivatives equal to zero. In this case, the stationary solution is given by [7,8]:

$$x_{is}^1 = 0, i = 1,...,n \tag{4}$$

In addition to the equilibrium point (4), system (3) admits other stationary solutions defined by

$$x_{is} = \sqrt[3]{\frac{b_n k_i - a_{n-i+1}}{2b_n}}, k_i^1 = \sqrt[3]{\left(\frac{b_n k_i - a_{n-i+1}}{2b_n} \right)^2}, i = 1,...,n \tag{5}$$

The stability of the equilibrium point (4) is investigated using a Lyapunov-based gradient approach [16,17]. In this framework, the behavior of the system is characterized through Lyapunov vector functions $V(x)$, which makes it possible to assess the convergence of trajectories in the state space.

The corresponding stability conditions are formulated in terms of inequalities imposed on the system parameters. In particular, the requirement of positive definiteness of the Lyapunov vector function leads to the following set of constraints [6,18]:

$$\begin{cases} b_n > 0 \\ -(b_n k_1 + a_n) \geq 0 \\ -(b_n k_2 + a_{n-1} + 1) \geq 0 \\ \dots \quad \dots \quad \dots \quad \dots \quad \dots \\ -(b_n k_n + a_1 + 1) \geq 0 \end{cases} \tag{6}$$

These inequalities define the parameter region in which the system exhibits stable behavior without oscillations.

To analyze the stability of the equilibrium point (5), the system is rewritten in terms of deviations from this state. This transformation allows the dynamics to be expressed in a form suitable for further analysis:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \vdots \\ \dot{x}_{n-1} = x_n \\ \dot{x}_n = b_n\left(-x_1^4 - 4x_{1s}x_1^3 - 3k_1^1x_1^2 - \left(k_1 - \dfrac{a_n}{b_n}\right)x_1\right) + \\ +b_n\left(-x_2^4 - 4x_{2s}x_2^3 - 3k_2^1x_2^2 - \left(k_2 - \dfrac{a_{n-1}}{b_n}\right)x_2\right) +,\dots,+ b_n\left(-x_n^4 - 4x_{ns}x_n^3 - 3k_n^1x_n^2 - \left(k_n - \dfrac{a_1}{b_n}\right)x_n\right) \end{cases} \tag{7}$$

The resulting system (7) is then examined within the same analytical framework, and the corresponding stability conditions can be obtained in an analogous manner [16,19].

The stability of the equilibrium point (5) requires the Lyapunov vector function to be positive definite. This leads to a set of constraints on the system parameters, which can be written as

$$\begin{cases} b_n > 0 \\ b_n k_1 - a_n \geq 0 \\ b_n k_2 - a_{n-1} - 1 \geq 0 \\ \dots \quad \dots \quad \dots \quad \dots \\ b_n k_n - a_1 - 1 \geq 0 \end{cases} \tag{8}$$

These constraints (8) define the parameter region in which the considered equilibrium state remains stable.

The following analysis focuses on the stability behavior of a self-organizing model with predefined dynamic properties.

A self-organizing model with specified parameters and desired transient characteristics can be represented by the following system of equations [16]:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \vdots \\ \dot{x}_{n-1} = x_n \\ \dot{x}_n = -x_1^4 - d_1^1x_1^2 + d_1x_1 - x_2^4 - d_2^1x_2^2 + d_2x_2 -,\dots,- x_n^4 - d_n^1x_n^2 + d_nx_n \end{cases} \tag{9}$$

The equilibrium solution of system (9) is given by

$$x_{is}^1 = 0, i = 1,...,n \tag{10}$$

Additional stationary solutions can be obtained in the form

$$x_{is} = \sqrt[3]{\frac{d_i}{2}}, d_i^1 = \sqrt[3]{\left(\frac{d_i}{2}\right)^2}, i = 1,...,n \tag{11}$$

The stability properties of the equilibrium state (10) are evaluated using Lyapunov-based analysis. In this case, the requirement of positive definiteness of the Lyapunov vector function leads to the following constraints on the system parameters:

$$-d_1 \geq 0, -\left(d_2 + 1\right) \geq 0, ..., -\left(d_n + 1\right) \geq 0, \tag{12}$$

These inequalities (12) define the region of parameter values for which the system remains stable and exhibits the desired transient behavior.

To examine the behavior of the system near the equilibrium point (11), the model is expressed in terms of deviations from this state. The resulting system can be written as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \vdots \\ \dot{x}_{n-1} = x_n \\ \dot{x}_n = b_n\left(-x_1^4 - 4x_{1s}x_1^3 - 3d_1^1x_1^2 - d_1x_1\right) + b_n(-x_2^4 - 4x_{2s}x_2^3 - 3d_2^1x_2^2 - d_2x_2) +, ..., + \\ +, ..., + b_n(-x_n^4 - 4x_{ns}x_n^3 - 3d_n^1x_n^2 - d_nx_n) \end{cases} \tag{13}$$

The stability properties of system (13) are evaluated within a Lyapunov-based framework [7,8]. The requirement of positive definiteness of the Lyapunov vector function $V(x)$ leads to a set of parameter constraints of the form

$$d_1 \geq 0, d_1 - 1 \geq 0, d_2 - 1 \geq 0, ..., d_n - 1 \geq 0, \tag{14}$$

These inequalities (14) determine the parameter region in which the equilibrium state (11) remains stable.

Combining the obtained conditions with relations (6) and (12), the coefficients of the controller can be expressed as

$$k_1 \geq \frac{d_1 + a_n}{b_n}, k_2 \geq \frac{d_2 + a_{n-1}}{b_n}, ..., k_n \geq \frac{d_n + a_1}{b_n} \tag{15}$$

The proposed synthesis procedure leads to a control system that ensures stable operation and satisfies the required performance characteristics.

Discussion

The obtained results demonstrate that the proposed self-organizing control system, constructed within the class of two-parameter structurally stable mappings, possesses well-defined regions of aperiodic robust stability. These regions are determined by the derived parameter inequalities, which impose constraints on both the system dynamics and the controller coefficients.

The analysis of stationary states shows that the stability properties of the system depend on admissible combinations of the control parameters. For the equilibrium corresponding to the nominal operating mode, the positivity conditions of the Lyapunov vector functions define a domain in which the system trajectories converge to the equilibrium point in a non-oscillatory manner. This implies that the transient response remains well damped and that stability is preserved under parametric uncertainty, in accordance with the derived stability conditions.

For additional stationary states, the obtained inequalities make it possible to identify parameter regions in which these equilibria also remain stable. This result demonstrates that the proposed framework allows explicit characterization of transitions between stationary states when the initial equilibrium loses robustness. In this context, the self-organizing effect is manifested through a change in the stability structure of the system and the emergence of new stable operating regimes.

The controller coefficients are determined directly from the derived stability conditions of both the control system and the corresponding self-organizing model with prescribed transient characteristics. As a result, the synthesis procedure ensures not only stability preservation but also the possibility of assigning desired transient performance within the admissible parameter region.

From an engineering perspective, the proposed method enables the construction of control laws that suppress oscillatory behavior and ensure convergence of the state trajectories under parametric uncertainty. This property is particularly important in practical applications, where even small parameter variations may significantly affect system performance.

To illustrate the theoretical results, a numerical experiment in MATLAB was carried out for a linear single-input single-output dynamic system under parametric uncertainty. The simulation was performed using a nonlinear control law belonging to the class of two-parameter structurally stable mappings.

Figure 1 shows the time responses of the state variables $x_1(t)$ and $x_2(t)$. Both trajectories converge to the trivial equilibrium within approximately 4–5 seconds. The transient response is well damped, with a small initial undershoot in $x_2(t)$, and no sustained oscillations are observed. This behavior confirms the stability of the system and its operation within the region of aperiodic robust stability.

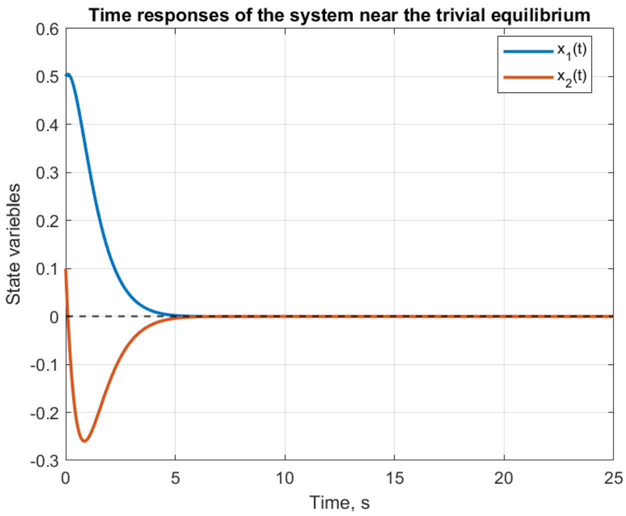

Figure 1. Time responses of the system near the trivial equilibrium

Figure 2 presents the phase trajectory in the (x_1, x_2) state space. The trajectory converges smoothly to the origin without forming closed loops or spiral patterns, which indicates the absence of oscillatory dynamics and confirms asymptotic stability of the trivial equilibrium.

A key result of the study is illustrated in Figure 3, which shows the bifurcation-induced transition from the trivial equilibrium to a nontrivial stationary state. Initially, the system operates in a regime where the trivial equilibrium is stable, and the state variables converge to zero.

At the moment indicated by the vertical dashed line, the system parameters are changed, leading to a loss of stability of the trivial equilibrium and the emergence of a new stable equilibrium.

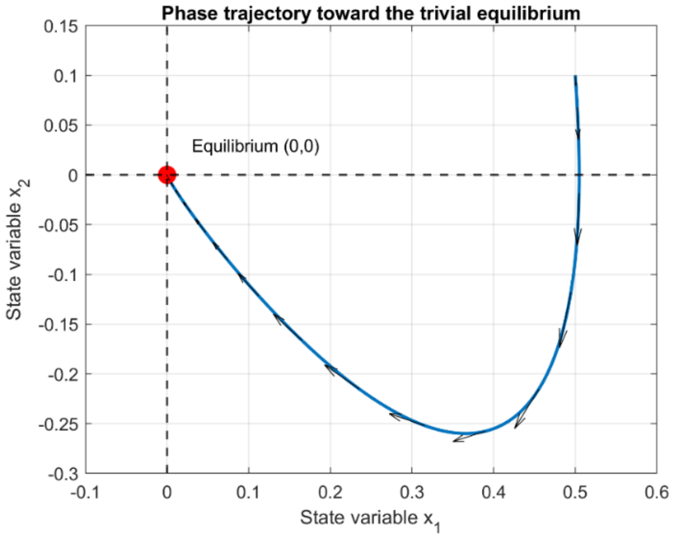

Figure 2. Phase trajectory toward the trivial equilibrium

As a result, the system transitions to a new operating regime, and the state variable $x_1(t)$ converges to a nonzero value corresponding to the nontrivial equilibrium. This behavior clearly demonstrates the self-organizing nature of the system.

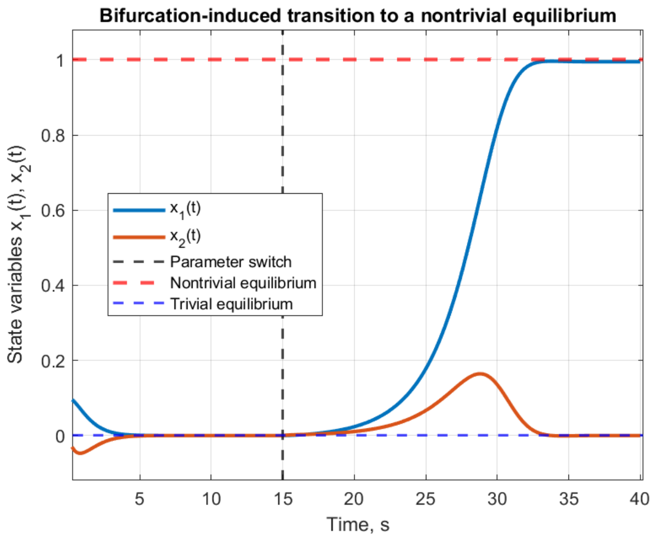

Figure 3. Bifurcation-induced transition to a nontrivial equilibrium

Figure 4 shows the phase trajectory near the nontrivial equilibrium. The system trajectories converge to a nonzero stationary point, indicating the stability of the new operating regime. The absence of oscillatory behavior confirms that the transient process remains well damped after the bifurcation.

Figure 5 illustrates the evolution of the Lyapunov function $V(t)$ for both the trivial and nontrivial equilibria. In both cases, the function decreases after a short initial transient and tends to zero, which is consistent with the theoretical stability conditions derived using the Lyapunov-based approach. The absence of oscillatory behavior in $V(t)$ further confirms that the system operates within the admissible stability region.

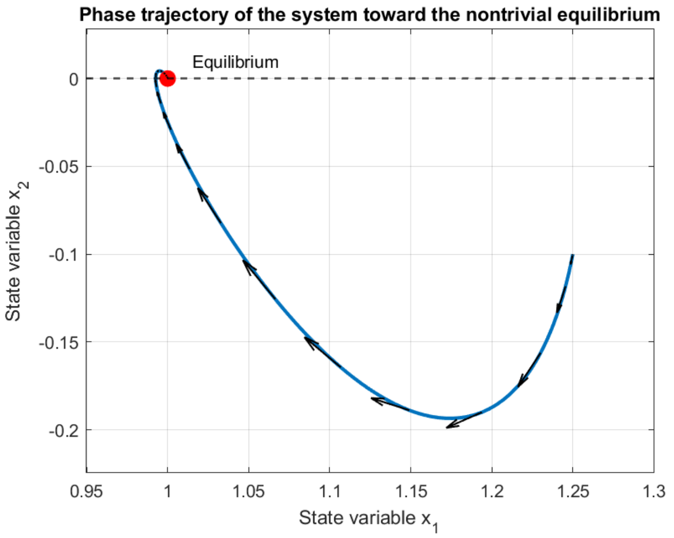

Figure 4. Phase trajectory of the system toward the nontrivial equilibrium

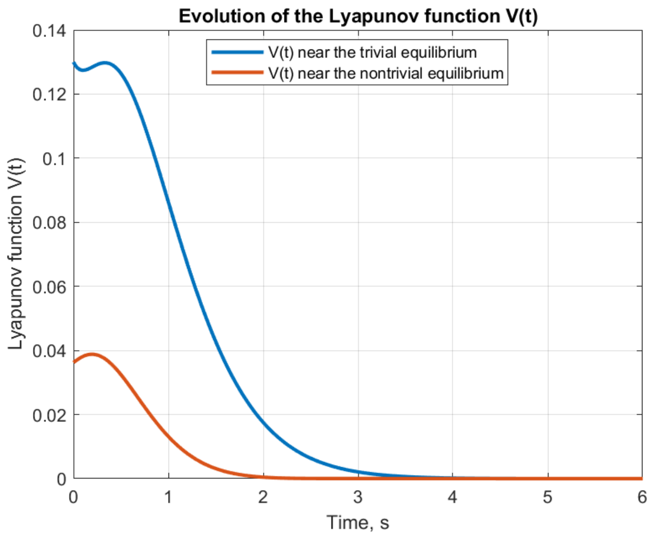

Figure 5. Evolution of the Lyapunov function $V(t)$

Thus, the numerical results are in full agreement with the theoretical analysis. The convergence of trajectories, the absence of oscillatory dynamics, and the observed transitions between stationary states confirm that the proposed method provides a consistent framework for the synthesis of self-organizing control systems with guaranteed stability and prescribed transient characteristics.

Conclusion

In this study, the problem of ensuring aperiodic robust stability in self-organizing control systems under parametric uncertainty is addressed. The analysis has been carried out within the framework of two-parameter structurally stable mappings for linear single-input single-output systems.

A set of conditions guaranteeing monotonic convergence of system trajectories has been obtained. These conditions are formulated in terms of inequalities derived from the properties of Lyapunov vector functions and define the admissible parameter regions ensuring stable operation of both the control system and its corresponding model.

Based on the obtained results, a procedure for determining the controller parameters has been developed. This procedure makes it possible to preserve aperiodic robust stability in the presence of uncertainty and to achieve the desired transient characteristics. The proposed approach provides a consistent framework for the synthesis and analysis of self-organizing control systems and can be effectively applied to the design of control strategies with improved robustness and performance.

The results demonstrate that the use of structurally stable mappings in combination with Lyapunov-based analysis provides an effective approach to the problem of stability under uncertainty. The derived conditions and synthesis procedure ensure non-oscillatory behavior within the admissible parameter region and extend the applicability of self-organizing control methods in engineering practice.

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