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CHARACTERISTIC FUNCTION OF THE SYSTEM D–EQUATIONS

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Abstract

This paper is devoted to the problems of studying the multiperiodic solution of some evolutionary equations. The article also discusses the existence and uniqueness of a multiperiodic solution with respect to vector functions for an evolutionary reduced equation. Studies have been conducted on the characteristic function of a certain system of the evolutionary equation. Some properties of the vector function are proved. They can be used in the further study of oscillatory bounded solutions of evolutionary equations. Based on the argumentation of the theorem on the existence and uniqueness of an almost multiperiodic solution of the specified system, considered using the method of shortening the characteristic function. All estimates of the characteristic function are based on the enhanced Lipschitz condition, first introduced by academician K. P. Persian. The results will also be useful in the study of periodic solutions of evolutionary equations of mathematical physics

Keywords: strengthened Lipschitz condition, truncated differential operator, Bellman - Gronwall lemma (B.-G.).

Аңдатпа

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D-ТЕҢДЕУЛЕР ЖҮЙЕСІНІҢ СИПАТТАУШЫ ФУНКЦИЯСЫ

Бұл жұмыс кейбір эволюциялық теңдеулердің көппериодты шешімін зерттеу мәселелеріне арналған. Сондай-ақ, мақалада эволюциалық қысқартылған теңдеу үшін векторлық функцияларға қатысты көппериодтылық шешімнің бар болуы мен жалғыздық мәселелері қарастырылады. Эволюциялық теңдеудің кейбір жүйесінің сипаттаушы функциясына қатысты зерттеулер жүргізілді. Вектор функцияның кейбір қасиеттері дәлелденді. Олар эволюциялық теңдеулердің тербелмелі шектеулі шешімдерін одан әрі зерттеуде қолданылуы мүмкін. Көрсетілген жүйенің дерлік көп периодты шешімінің бар болуы және жалғыздығы туралы теореманың дәлелдеуіне сүйене отырып сипатттаушы функциянын бағамдары қысқарту әдісін пайдалана отырып қарастырылған. Сипаттаушы функцияның барлық бағалары алғаш рет академик К.П.Персидский енгізген Липшицтің күшейтілген шартына негізделген. Нәтижелерді математикалық физиканың эволюциялық теңдеулерінің периодты шешімдерін зерттеуде де пайдалы болады

Түйін сөздер: Липшицтің күшейтілген шарты, қысқартылған дифференциалдау операторы, Беллман - Гроноулл леммасы

Аннотация

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ХАРАКТЕРИСТИЧЕСКАЯ ФУНКЦИЯ СИСТЕМЫ D-УРАВНЕНИЙ

Данная работа посвящена проблемам изучения многопериодического решения некоторых эволюционных уравнений. В статье также рассматриваются вопросы существования и единственности многопериодического решения относительно векторных функций для эволюционного сокращенного уравнения. Проведены исследования относительно характеристической функции некоторой системы эволюционного уравнения. Доказаны некоторые свойства векторной функции. Они могут быть использованы при дальнейшем изучении колебательных ограниченных решений эволюционных уравнений. Исходя из аргументации теоремы о существовании и единственности почти многопериодического решения указанной системы, рассмотренные с использованием метода укорочения характеристической функции. Все оценки характеристической функции основаны на усиленном условие Липшица, впервые введенном академиком К.П. Персидским. Результаты также будут полезны при изучении периодических решений эволюционных уравнений математической физики

Ключевые слова: усиленное условие Липшица, укороченный оператор дифференцирования, лемма Беллмана - Гронулла (Б.-Г.).

If $t \in R = (-\infty, \infty)$, $\varphi \in R_\varphi = \{\varphi : \|\varphi\| < \infty\}$, $\varphi = (\varphi_1, \varphi_2, ..., \varphi_m, ...)$ is a countable vector, with norm

$$\|\varphi\| = \sup_k |\varphi_k|.$$

In this case, we write

$$W_m \varphi = (\varphi_1, ..., \varphi_m, 0, 0, ...).$$

Consider the differential equation

$$\frac{d\varphi}{dt} = a[t, W_m \varphi, f(t, W_m \varphi), \mu] \cdot \tag{1}$$

The condition (π_1^∞) ensures the existence and uniqueness of the solution to the Cauchy problem for equation (1) with respect to vector functions $a(t, W_m \varphi, . f(t, W_m \varphi), \mu)$

Let us denote $\varphi = \xi_m(t, t_0, \varphi_0)$ by the solution of equation (1) passing through the point $(t_0, \varphi_0) \in R \times R_\varphi$ .Solving equations (1) with respect φ_0 to obtain a characteristic function $\xi_m(t, t_0, \varphi_0)$ that admits an integral representation in the form

$$\xi_m(t_0, t, \varphi) = W_m \varphi + \int_t^{t_0} W_m a[s, \xi_m, f(s, \xi_m), \mu] ds \cdot \tag{2}$$

Here are some properties of the vector - function $\xi_m(t, t_0, \varphi_0)$.

1[0]. The vector function $\xi_m(t, t_0, \varphi_0)$ for any fixed $t_0 \in R$ satisfies the equation

$$D\xi_m(t_0, t, \varphi) = 0.$$

Evidence. It is known that it $\lambda_f(t_0, t, \varphi)$ belongs to the π - class, and it is continuously differentiable by t [1-3].

Similarly, this expression is suitable for the characteristic vector function $\xi_m(t, t_0, \varphi_0)$ of the truncated differential operator

$$D_m = \frac{\partial}{\partial t} + \sum_{k=1}^{m} W_m a_k(t, W_m \varphi, \gamma, \mu) \frac{\partial}{\partial \varphi_k} \cdot$$

Therefore, the function $\xi_m(t_0, \tau, \xi_m(\tau, t, \varphi))$ is differentiable with respect τ to a parameter, and

$$[\xi_m(t_0,\tau,\xi_m(\tau,t,\phi))]'_\tau = \frac{\partial \xi_m(t_0,\tau,\xi_m(\tau,t,\phi))}{\partial t} + \frac{\partial \xi_m(t_0,\tau,\xi_m(\tau,t,\phi)}{\partial \phi_k}\cdot$$

$$\frac{\partial \xi_m(\tau,t,\phi)}{\partial t} = \frac{\partial \xi_m(t_0,\tau,\xi_m(\tau,t,\phi))}{\partial t} + \sum_{k=1}^{m} W_m a_k(\tau,\xi_m(\tau,t,\phi))\cdot$$

$$\frac{\partial \xi_m(t_0,\tau,\xi_m(\tau,t,\phi))}{\partial \phi_k} = D\xi_m(t_0,\tau,\xi_m(\tau,t,\phi))$$

But since we have $\xi_m(t_0,\tau,\xi_m(\tau,t,\varphi)) = \xi_m(\tau_0,t,\varphi)$, the considered function is constant with respect to the parameter τ . Therefore

$$D\xi_m(t_0,\tau,\xi_m(\tau,t,\varphi)) \equiv D\xi_m(\tau_0,t,\varphi) = 0 \text{, therefore}$$

$$D\xi_m(\tau_0,t,\varphi) = 0 .$$

2[0]. The vector function $\xi_m(\tau_0,t,\varphi)$ has continuous partial derivatives with respect to all arguments [4].

Evidence. The existence of the derivative with respect to t is obvious, because

$$\frac{d\xi_m(\tau_0,t,\varphi)}{dt} = W_m a[s,\xi_m, f(s,\xi_m),\mu]\cdot$$

The derivatives with respect to t,φ_k the equations in variations

$$\frac{d}{dt}\left(\frac{d\xi_m(t_0,t,\varphi)}{\partial t}\right) = \frac{\partial W_m a[t,\xi_m, f(t,\xi_m),\mu]}{\partial \varphi}\cdot\frac{\partial \xi_m(t_0,t,\varphi)}{\partial t},$$

$$\frac{d}{dt}\left(\frac{d\xi_m(t_0,t,\varphi)}{\partial t}\right) = \frac{\partial W_m a[t,\xi_m, f(t,\xi_m),\mu]}{\partial \varphi}\cdot\frac{\partial \xi_m(t_0,t,\varphi)}{\partial \varphi_k}.$$

Thus, to calculate the indicated derivatives, we obtain linear equations in variations for the solution $\xi_m(\tau_0,t,\varphi)$. Since these equations are linear, their solutions always exist and are continuous. For these derivatives, we obtain the estimates

$$\left\|\frac{\partial \xi_m(t_0,t,\varphi)}{\partial t}\right\| \le r_0 e^{r_0|t-t_0|},$$

$$\left\|\frac{\partial \xi_m(t_0,t,\varphi)}{\partial \varphi_k}\right\| \le e^{r_0|t-t_0|}.$$

3[0]. Directly from (2), on the basis of the strengthened Lipschitz condition and the boundedness of the vector function, $a(t,W_m\varphi,\gamma,\mu)$ we have

$$\|\xi_m(t_0,t,\varphi)\| \le \|W_m\varphi\| e^{r_0|t-t_0|} + \frac{\bar{a}_0}{r_0}(e^{r_0|t-t_0|}-1)\cdot$$

Evidence.

$$\left\|\xi_m(t_0,t,\varphi)\right\| \le \left\|W_m\varphi\right\| + \left|\int_t^{t_0}\left\|W_m a\left[s,\xi_m, f(s,\xi_m),\mu\right]\right\|ds\right| \le$$

$$\le \left\|W_m\varphi\right\| + \left|\int_t^{t_0}\left\{\left\|W_m\left\{a(s,\xi_m(s,t,\varphi), f(s,\xi_m(s,t,\varphi)),\mu) - a(s,0,f(s,0),\mu)\right\}\right\| + \right.\right.$$

$$\left.\left. + \left\|W_m\left\{a(s,0,f(s,0),\mu) - a(s,0,0,\mu)\right\}\right\| + \left\|W_m a(s,0,0,\mu)\right\|\right\}ds\right| \le \left\|W_m\varphi\right\| +$$

$$+\left|\int_t^{t_0}\left\{(\alpha_0+\bar{\alpha}_1\delta_0)\left\|W_m\xi_m(s,t,\varphi)\right\| + \bar{\alpha}_1\left\|W_m f(s,0)\right\| + W_m a_0\right\}ds\right| \le$$

$$\le \left\|W_m\varphi\right\| + \left|\int_t^{t_0}\left\{(\alpha_0+\bar{\alpha}_1\delta_0)\left\|\xi_m(s,t,\varphi)\right\| + W_m(\bar{\alpha}_1\Delta + a_0)\right\}ds\right| \le$$

$$\le \left\|W_m\varphi\right\| + \left|\int_t^{t_0}\left[r_0\left\|\xi_m(s,t,\varphi)\right\| + \bar{a}_0\right]ds\right|.$$

Therefore

$$\left\|\xi_m(t_0,t,\varphi)\right\| \le \left\|W_m\varphi\right\| e^{r_0|t-t_0|} + \frac{\bar{a}_0}{r_0}\left(e^{r_0|t-t_0|}-1\right) [5].$$

<u>LEMMA 1.</u> For any natural number m , the following estimates are valid:

1 а) $\left\|\xi_m(t_0,t,\bar{\varphi}) - \xi_m(t_0,t,\varphi)\right\| \le \left\|W_m(\bar{\varphi}-\varphi)\right\| e^{r_0|t-t_0|}$,

1 б) $\left\|\xi_m(t_0+\tau,t+\tau,\bar{\varphi}+\theta) - \xi_m(t_0,t,\varphi) - W_m\theta\right\| \le \frac{\left\|\Delta_{\tau,\theta}{}^r\right\|}{r_0}\left(e^{r_0|t-t_0|}-1\right)$,

1 в) $\left\|\xi_{m_f}(t_0,t,\varphi) - \xi_{m_g}(t_0,t,\varphi)\right\| \le \frac{\bar{\alpha}_1\left\|W_m(f-g)\right\|}{r_0}\left(e^{r_0|t-t_0|}-1\right)$.

Evidence.
1 а) [6]

$$\left\|\xi_m(t_0,t,\bar{\varphi}) - \xi_m(t_0,t,\varphi)\right\| \le \left\|W_m(\varphi)\right\| +$$

$$+\left|\int_t^{t_0}\left\|W_m a[s,\xi_m(s,t,\bar{\phi}), f(s,\xi_m(s,t,\bar{\phi})),\mu] - W_m a[s,\xi_m(s,t,\phi), f(s,\xi_m(s,t,\phi)),\mu]\right\|ds\right| \le$$

$$\leq \left\| W_m\left(\bar{\phi}-\phi\right)\right\| +$$

$$+\left|\int_t^{t_0}\left\| W_m\left\{a[s,\xi_m(s,t,\bar{\phi}), f(s,\xi_m(s,t,\bar{\phi})),\mu]-a[s,\xi_m(s,t,\phi), f(s,\xi_m(s,t,\phi)),\mu]\right\}\right\| ds\right| \leq$$ By

$$\leq \left\| W_m\left(\bar{\phi}-\phi\right)\right\| + \left|\int_t^{t_0} W_m\left(r_0\left\|\xi_m(s,t,\bar{\phi})-\xi_m(s,t,\phi)\right\|\right)ds\right|.$$

lemma B.-G. [1] we get

$$\left\|\xi_m(t_0,t,\overline{\varphi})-\xi_m(t_0,t,\varphi)\right\| \leq \left\| W_m(\overline{\varphi}+\varphi)\right\| e^{r_0\left|t-t_0\right|}.$$

1 б)

$$\left\|\xi_m(t_0+\tau,t+\tau,\phi+\theta)-\xi_m(t_0,t,\phi)-W_m\theta\right\| \leq$$

$$\leq \left|\int_t^{t_0}\left\{\left\| W_m r_0\left\{\xi_m(s+\tau,t+\tau,\phi+\theta)-\xi_m(s,t,\phi)-W_m\theta\right\}\right\| + \left\| W_m\Delta_{\tau,\theta}r\right\|\right\}ds\right|.$$

Applying the lemma of B.-G. we have [7]

$$\left\|\xi_m(t_0+\tau,t+\tau,\varphi+\theta)-\xi_{m_g}(t_0,t,\varphi)-W_m\theta\right\| \leq \frac{\left\|\Delta_{\tau,\theta}r\right\|}{r_0}\left(e^{r_0\left|t-t_0\right|}-1\right).$$

1 в)

$$\left\|\xi_{m_f}(t_0,t,\phi)-\xi_{m_g}(t_0,t,\phi)\right\| \leq$$

$$\leq \left|\int_t^{t_0}\left\{\bar{\alpha}_1\left\| W_m(f-g)\right\|_H + (\alpha_0+\bar{\alpha}_1\delta_0)\left\|\xi_{m_f}-\xi_{m_g}\right\|\right\}ds\right|.$$

By lemma B.-G. get

$$\left\|\xi_{m_f}(t_0,t,\varphi)-\xi_{m_g}(t_0,t,\varphi)\right\| \leq \frac{\bar{\alpha}_1\left\| W_m(f-g)\right\|_H}{r_0}\left(e^{r_0\left|t-t_0\right|}-1\right).$$

It is further shown that an almost multiperiodic solution of the basic system can be uniformly approximated by an almost multiperiodic solution of the system in the form μ :

$$D_m^y y = P(t,W_m\varphi)y+\mu Q\{t,W_m\varphi,y,\mu)+\mu\int_{-\infty}^{\infty}R[t_1,t,W_m\varphi,y,\mu]\psi(t-t_1)dt_1 \quad [8].$$

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