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REFERENCE MODEL SELECTION OF THE ADAPTIVE CONTROL SYSTEM FOR AN OBJECT WITH SINGLE-INPUT AND SINGLE-OUTPUT

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Abstract

One of the most promising ways to solve the problem of control in the conditions of uncertainty of the characteristics of the control object and external influences is the use of adaptation methods. The article proposes to investigate the asymptotic robust stability of the reference model with the desired dynamics, using the gradient-velocity method of the Lyapunov vector-function. Considering control systems as gradient systems, and Lyapunov functions as potential functions allowed us to develop a universal gradient-velocity method of the Lyapunov vector-function, which allows to investigate the aperiodic robust stability of the reference model by the desired dynamics.

Key words: adaptive control, gradient-velocity method of the Lyapunov vector-function, object with single-input and single-output.

Аңдатпа

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БІР КІРІС ЖӘНЕ БІР ШЫҒЫСЫ БАР ОБЪЕКТ ҮШІН АДАПТИВТІ БАСҚАРУ ЖҮЙЕСІНІҢ ЭТАЛОНДЫҚ МОДЕЛІН ТАҢДАУ

Басқару объектісінің сипаттамалары мен сыртқы әсерлердің белгісіздігі жағдайларында басқару мәселесін шешудің ең перспективті әдістерінің бірі – адаптация әдістерін қолдану. Мақалада қажетті динамикасы бар эталондық модельдің асимптотикалық робасты орнықтылығын Ляпуновтың градиенті-жылдамдықтық вектор-функциясы тәсілімен зерттеу ұсынылады. Басқару жүйесін градиенттік жүйелер ретінде, ал Ляпуновтың функцияларын потенциалды функциялар ретінде қарастыру эталондық модельдің апериодтық робасты орнықтылығын қажетті динамикамен зерттеуге мүмкіндік беретін әмбебап Ляпуновтың градиенті-жылдамдықтық вектор-функциясы тәсілімен жасауға мүмкіндік берді.

Түйін сөздер: адаптивті басқару, Ляпуновтың градиенті-жылдамдықтық вектор-функциясы тәсілі, бір кірісі және бір шығысы бар объект.

Аннотация

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ВЫБОР ЭТАЛОННОЙ МОДЕЛИ АДАПТИВНОЙ СИСТЕМЫ УПРАВЛЕНИЯ ДЛЯ ОБЪЕКТА С ОДНИМ ВХОДОМ И ОДНИМ ВЫХОДОМ

Одним из наиболее перспективных путей решения проблемы управления в условиях неопределенности характеристик объекта управления и внешних воздействий является применение методов адаптации. В статье предлагается исследовать асимптотической робастной устойчивости эталонной модели с желаемой динамикой, градиентно-скоростным методом вектор-функции Ляпунова. Рассмотрение системы управления как градиентные системы, а функций Ляпунова, как потенциальные функции позволило разработать универсальный градиентно-скоростной метод вектор-функции Ляпунова, который позволяет исследовать апериодической робастной устойчивости эталонной модели желаемой динамикой.

Ключевые слова: адаптивное управление, градиентно-скоростной метод вектор-функции Ляпунова, объект с одним входом и одним выходом.

Introduction

The current fourth industrial revolution involves the widespread creation and use of automatic and automated control systems. Automatic control systems (ACS) are used in almost all branches of production and technology: in mechanical engineering, energy, electronics, chemical, biological, metallurgical and textile industries, transport, robotics, aviation, space systems, high-precision military equipment and technology, etc. However, to solve a wide range of applied problems, traditional methods of designing and researching ACS, which require knowledge of an adequate mathematical model of the object, are unacceptable. The higher the quality of traditional (non-adaptive) control methods, the more a priori information about the object itself and the conditions of its functioning. In practice, it is quite difficult to provide an accurate mathematical description of the control object. Moreover, the characteristics of the object and external influences in the process of functioning can change significantly. Under these conditions, traditional methods are often not applicable or do not provide the required quality of the automatic control system.

One of the most promising ways to solve this problem is the use of adaptation methods. In adaptive control systems, external influences are compensated, i.e. the control system becomes invariant with respect to external influences, and information about the object is collected during operation, immediately processed and used to generate control actions. This allows it possible to improve the quality of control under conditions of uncertainty of the parameters of the object and the operating environment.

We consider self- adjusting systems, where the structure of the control object is known and unchanged, and the behavior depends on a number of unknown parameters. This problem is solved in the class of self-adjusting systems (SAS), in which the structure of the regulator is set (pre-selected) and it is only necessary to determine the algorithm for adjusting its coefficients (the adaptation algorithm).

SAS are divided into with search and without search. We consider SAS without search, where there is an explicit or implicit model with the desired dynamic characteristics. The task of the adaptation algorithm is to adjust the coefficients of the regulator in such a way as to reduce the mismatch between the control object and the model to zero. Such control is called direct adaptive control, and systems are called adaptive systems with a reference model.

Methods for the synthesis of adaptation algorithms are developing in several directions [4,5]: gradient methods [4,5,6], methods based on the application of the Lyapunov function [4,5,6,7], methods based on the theory of hyperstability [12], methods based on the organization of sliding modes [6], methods based on the introduction of an “infinitely large” gain [8].

The synthesis of adaptation algorithms based on the gradient-velocity method of Lyapunov vector-functions [1,2,3,9,13] in the framework of the method based on the application of the Lyapunov function is a new scientific direction. The gradient-velocity method of the Lyapunov vector-function allows us to propose a universal approach to the construction of the Lyapunov function. In this case, the control system is considered as gradient systems and the Lyapunov function as potential functions from the theory of catastrophes [10, 11], and the gradient condition of the control system allows us to uniquely and analytically construct the required Lyapunov functions. The necessary condition for the existence of Lyapunov functions corresponds to the asymptotic, robust stability of the ACS with the desired dynamics, and on this basis, the main goal of adaptive control is always achievable.

This article is devoted to the formulation of the adaptive control problem and the study of the aperiodic robust stability of the reference model of the system by the gradient-velocity method of the Lyapunov vector-function.

Material and Methods

Let us consider a linear time-invariant control system [1,3,4]

$$\dot{x} = Ax + Bu, \tag{1}$$

where $x(t) \in R^n$ – state vector of control object (CO); $u(t) \in R^m$ – vector of control, $A \in R^{n\times n}$ matrix of the control object, $B \in R^{n\times m}$ constant matrix of control. It is assumed that the entire state vector of control object is available for measurement, thus $y(t) = x(t)$.

The problem of CO provision with desired dynamics, which will be set by reference model, is also considered

$$\dot{x}_M = A_M x_M + B_M r(t), \tag{2}$$

where $x_M(t) \in R^n$ – state vector of reference model; $r(t) \in R^m$– control input.

The reference model selection depends on requirements to closed system (transient response time, overshoot period, steady-state error, etc.). Moreover, it must be stable i.e. suppose the reference model should be aperiodic, robust and stable.

We formalize the control goal, by requiring that

$$\lim_{t\to\infty} \varepsilon(t) = 0 \tag{3}$$

where $\varepsilon(t) = x(t) - x_M(t)$ – system error (1) and (2).

Therefore, the problem of adaptive system construction with explicit reference model is set. We solve the problem on the basis of direct adaptive approach. In accordance with two-level structure of SAS, the problem will be solved in two steps: main circuit construction and adaptation circuit synthesis.

Results and discussion

Synthesis of the main circuit. To obtain the structure of the "ideal" regulator, we write down the equations in deviations

$$\dot{\varepsilon} = A_M\varepsilon + (A - A_M)x + Bu - B_M r, \tag{4}$$

The condition for the decidability of equation (4) is

$$(A - A_M)x + Bu - B_M r=0, \tag{5}$$

With respect to $u_* \in R^m$ for any $x(t) \in R^n$, $r(t) \in R^m$. this case, equation (4) will have the form

$$\dot{\varepsilon} = A_M\varepsilon, \tag{6}$$

The control goal (3) is achieved when the conditions (5) and the aperiodic robust stability of the system (6) are fulfilled.

The system (6) is investigated on the aperiodic robust stability by the gradient-velocity method of the Lyapunov vector-functions [1,9,13], where

$$A_M = \begin{Vmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 \\ -d_n & -d_{n-1} & -d_{n-2} & \cdots & -d_1 \end{Vmatrix}$$

System (6) is presented in an expanded form

$$\begin{cases} \dot{\varepsilon}_1 = \varepsilon_2 \\ \dot{\varepsilon}_2 = \varepsilon_3 \\ \quad\cdots \\ \dot{\varepsilon}_{n-1} = \varepsilon_n \\ \dot{\varepsilon}_n = -d_n\varepsilon_1 - d_{n-1}\varepsilon_2 - d_{n-2}\varepsilon_3, \dots, -d_1\varepsilon_n \end{cases} \tag{7}$$

From equation (7), we determine the components of the gradient vector from the Lyapunov function $V(\varepsilon) = \left(V_1(\varepsilon), V_2(\varepsilon), \dots, V_n(\varepsilon)\right)$:

$$\begin{cases} \frac{\partial V_1(\varepsilon)}{\partial \varepsilon_1} = 0, \quad \frac{\partial V_1(\varepsilon)}{\partial \varepsilon_2} = -\varepsilon_2, \quad \frac{\partial V_1(\varepsilon)}{\partial \varepsilon_3} = 0, \dots, \frac{\partial V_1(\varepsilon)}{\partial \varepsilon_n} = 0 \\ \frac{\partial V_2(\varepsilon)}{\partial \varepsilon_1} = 0, \quad \frac{\partial V_2(\varepsilon)}{\partial \varepsilon_2} = 0, \quad \frac{\partial V_2(\varepsilon)}{\partial \varepsilon_3} = -\varepsilon_3, \dots, \frac{\partial V_2(\varepsilon)}{\partial \varepsilon_n} = 0 \\ \dots \quad \dots \quad \dots \quad \dots \quad \dots \\ \frac{\partial V_{n-1}(\varepsilon)}{\partial \varepsilon_1} = 0, \quad \frac{\partial V_{n-1}(\varepsilon)}{\partial \varepsilon_2} = 0, \quad \frac{\partial V_{n-1}(\varepsilon)}{\partial \varepsilon_3} = 0, \dots, \frac{\partial V_{n-1}(\varepsilon)}{\partial \varepsilon_n} = -\varepsilon_n \\ \frac{\partial V_n(\varepsilon)}{\partial \varepsilon_1} = d_n \varepsilon_1, \quad \frac{\partial V_n(\varepsilon)}{\partial \varepsilon_2} = d_{n-1} \varepsilon_2, \quad \frac{\partial V_n(\varepsilon)}{\partial \varepsilon_3} = d_{n-1} \varepsilon_3, \dots, \frac{\partial V_n(\varepsilon)}{\partial \varepsilon_n} = d_1 \varepsilon_n \end{cases} \tag{8}$$

From (7) we determine the components of the decomposition of the velocity vector by the coordinates of the system $(\varepsilon_1, \cdots, \varepsilon_n)$:

$$\begin{cases} \left(\frac{d\varepsilon_1}{dt}\right)_{\varepsilon_1} = 0, \left(\frac{d\varepsilon_1}{dt}\right)_{\varepsilon_2} = \varepsilon_2, \left(\frac{d\varepsilon_1}{dt}\right)_{\varepsilon_3} = 0, \dots, \left(\frac{d\varepsilon_1}{dt}\right)_{\varepsilon_n} = 0 \\ \left(\frac{d\varepsilon_2}{dt}\right)_{\varepsilon_1} = 0, \left(\frac{d\varepsilon_2}{dt}\right)_{\varepsilon_2} = 0, \left(\frac{d\varepsilon_2}{dt}\right)_{\varepsilon_3} = \varepsilon_3, \dots, \left(\frac{d\varepsilon_2}{dt}\right)_{\varepsilon_n} = 0 \\ \dots \quad \dots \quad \dots \quad \dots \quad \dots \\ \left(\frac{d\varepsilon_{n-1}}{dt}\right)_{\varepsilon_1} = 0, \left(\frac{d\varepsilon_{n-1}}{dt}\right)_{\varepsilon_2} = 0, \left(\frac{d\varepsilon_{n-1}}{dt}\right)_{\varepsilon_3} = 0, \dots, \left(\frac{d\varepsilon_{n-1}}{dt}\right)_{\varepsilon_n} = \varepsilon_n \\ \left(\frac{d\varepsilon_n}{dt}\right)_{\varepsilon_1} = -d_n\varepsilon_1, \left(\frac{d\varepsilon_n}{dt}\right)_{\varepsilon_2} = -d_{n-1}\varepsilon_2, \left(\frac{d\varepsilon_n}{dt}\right)_{\varepsilon_3} = -d_{n-2}\varepsilon_3, \dots, \left(\frac{d\varepsilon_n}{dt}\right)_{\varepsilon_n} = -d_1\varepsilon_n \end{cases} \tag{9}$$

The total time derivative of the Lyapunov vector functions is defined as the scalar product of the gradient vector (8) by the velocity vector (9)

$$\frac{dV(\varepsilon)}{dt} = -\varepsilon_2^2 - \varepsilon_3^2 -, \cdots, -\varepsilon_n^2 - (d_n\varepsilon_1)^2 - (d_{n-1}\varepsilon_2)^2 - (d_{n-2}\varepsilon_3)^2 -, \cdots, -(d_1\varepsilon_n)^2, \tag{10}$$

It follows from (10) that the total time derivative of the Lyapunov vector-function is guaranteed to be a negative function, i.e. the sufficient condition for asymptotic stability is guaranteed to be fulfilled.

The Lyapunov function in scalar form by components of the gradient vector (8) can be represented as:

$$V(\varepsilon) = \frac{1}{2} d_n \varepsilon_1^2 + \frac{1}{2}(d_{n-1} - 1)\varepsilon_2^2 + \frac{1}{2}(d_{n-2} - 1)\varepsilon_3^2 +, \dots, + \frac{1}{2}(d_1 - 1)\varepsilon_n^2, \tag{11}$$

From (11) the condition of aperiodic robust stability of the reference model is obtained in the form

$$d_n > 0, d_{n-1} - 1 > 0, d_{n-2} - 1 > 0, \dots, d_1 - 1 > 0. \tag{12}$$

The system of inequalities (12) is a necessary condition for the positive definiteness of the quadratic form (11), i.e., the condition for the existence of the Lyapunov vector-function, and is a condition for the aperiodic robust stability of the reference model by the desired dynamics. The reference model with the desired dynamics is characterized by a set of quality indicators, such as stability, robustness, oscillation, speed, lack of overshoot, static accuracy, the desired type of transients, etc.

Conclusion

The adaptive control system becomes invariant with respect to external influences, and information about the object during operation is processed, and the system generates a control action. All this makes it possible to improve the quality of management in the conditions of uncertainty of the parameters of the object and the operating environment. The main methods of synthesis of adaptation algorithms are based on the construction of the Lyapunov function. But at present, there are no universal approaches to the construction of Lyapunov functions and, accordingly, methods for the synthesis of adaptation algorithms.

Considering the control system as gradient systems, and the Lyapunov functions as potential functions, allowed us to develop a universal gradient-velocity method of the Lyapunov vector function, which allows us to study the aperiodic robust stability of the reference model with the desired dynamics. The reference model can be selected based on a set of quality indicators, such as stability, robustness, oscillation, speed, lack of overshoot, static accuracy, the desired type of transient processes in the system, etc.

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