ECONOMETRIC DYNAMIC INVESTMENT MODEL ON THE EXAMPLE OF THE DATA OF THE REPUBLIC OF KAZAKHSTAN

Babich V.V.

Abstract

The article provides the analysis of one of the most significant macroeconomic categories that affect the country's economic growth - gross investment. The author, on the basis of statistical information for the period of 1993-2020, determines the dependence of current investments on investments of the previous period and the growth rate of real gross domestic product. The sensitivity coefficients of changes in investments from accumulated investments of the previous period and the dynamics of national income are determined. An analysis is made of the necessary adequacy level of autonomous and induced investments which maintain an increase of the rate of economic growth. An autoregressive dynamic model is considered. A regression equation is constructed that gives estimates of the nonlinear investment function. For the composition, the author uses the method of partial correction, which eliminates such a problem as the bias of the estimates obtained.

Keywords: economic model, macroeconomics, economic growth, investment, autoregressive dynamic model, multicollinearity, analysis of variance, hypothesis testing.

Аннотация

В.В. Бабич

Эконометрическая динамическая модель инвестиций на примере данных Республики Казахстан

В статье рассматривается анализ одной из самых существенных макроэкономических категорий, оказывающих воздействие на экономический рост страны — валовых инвестиций. Автор на основании
Инвестиции являются одним из ключевых факторов, определяющих изменение темпов экономического роста в стране. Они могут быть автономными и индуцированными. Автономные инвестиции являются неотъемлемой частью любого экономического моделирования. Они включаются в модель через постоянное обновление капитала, что связано с постоянными технологическими изменениями. Однако, если экономика подвергается внешним шокам, связанным с неожиданными катализаторами, например, пандемией или политическими событиями, которые могут привести к изменению ключевых макроэкономических индикаторов, автономные инвестиции могут стать недостаточными для компенсации уменьшения темпов экономического роста. В этом случае, индуцированные инвестиции необходимы.

Индуцированные инвестиции предназначены для увеличения темпов экономического роста. Как правило, они проводятся в соответствии с принятыми стратегическими целями развития экономики страны. Инвестиционные аналитики считают, что инвестиции являются самым вольтменным компонентом фундаментальной макроэкономической идентичности [1]. Необходимо отметить, что мы рассматриваем инвестиции в реальной экономике, инвестиции в ценные бумаги, которые являются альтернативой сбережениям, не рассматриваются.

Известно, что инвестиции текущего периода пропорциональны изменению объема производства, измеряемого на основе динамики валового внутреннего продукта и уровня инвестиций предшествующего периода. По ускорительной парадигме, оптимальный запас инвестиционного капитала пропорционален объему ожидаемого производства. Любое увеличение ожидаемого реального валового внутреннего продукта в текущем периоде требует увеличения инвестиций в предшествующий период. Однако в краткосрочном периоде ограничивающим фактором является проявление принципа монотонной убывающей отдачи, что снижает макропродуктивность капитала и может сделать инвестиции неэффективными.

Таким образом, теория инвестиций динамична в своем характере, поскольку она связывает настоящее с будущим, поэтому стимулирование инвестиций в текущий момент увеличивает валовой внутренний продукт в будущем [2]. Модели, которые прямо участвуют в учете фактора времени, обычно называются динамическими. В таких моделях, все переменные экономических процессов и систем являются функциями времени [3].

Гипотеза и модель

Как было описано выше, инвестиции в текущем периоде зависят от индекса реального валового внутреннего продукта (ВВП) и уровня инвестиций предыдущих периодов.

Нулевая гипотеза: нет зависимости инвестиций в текущем периоде от инвестиций в предыдущем периоде и ВВП. Для тестирования гипотезы и анализа взаимосвязи между изучаемыми экономическими категориями, мы используем эмпирическую модель. Необходимо отметить, что выбор модели включает в себя множество вариантов и решений относительно длины пробы, преобразования переменных, порядка их динамики и других [4].

Гипотеза будет тестироваться на основе данных по основным макро-экономическим показателям Казахстана за период 1993-2020 на квартальной основе. Поскольку мы используем временные ряды, необходимо проверить эти ряды на стационарность. Рассмотрите тренд инвестиций, показанный на рис. 1. Анализ графика показывает, что ряд не стационарен в следующих значениях: в каждом периоде времени, средние изменения и амплитуда колебаний в значениях инвестиций в течение периодов различны.
To check the series for stationarity, we use the extended Dickey-Fuller test. The test results are shown in Model1.

Model1: Extended Dickey-Fuller test for the dependent variable

Extended Dickey-Fuller Test for Investment
Test starting from 12 lags, AIC criterion
Sample size 100
Unit root null hypothesis: \( a = 1 \)

**Constant test**
including 11 lag(s) for (1-L)Investment
model: \((1-L)y = b0 + (a-1)*y(-1) + ... + e\)
score for \((a-1): 0.0443686\)
test statistic: \( \tau_c(1) = 2.29039 \)
asympt. p-value 1
coefficient 1st order autocorrelations for e: 0.003
difference lag: \( F(11, 87) = 829.828 \ [0.0000] \)

**With constant and trend**
including 11 lag(s) for (1-L)Investment
model: \((1-L)y = b0 + b1*t + (a-1)*y(-1) + ... + e\)
score for \((a-1): -0.0344929\)
test statistic: \( \tau_{ct}(1) = -0.741804 \)
asympt. p-value **0.9692**
coefficient 1st order autocorrelations for e: -0.005
difference lag: \( F(11, 86) = 561.608 \ [0.0000] \)

The test data allow us to conclude that the series is integrated. Therefore, we will carry out a similar test for the first differences of the variable under consideration (investment). The test results are shown in Model 2. Analysis of Model 2 allows us to conclude that the null hypothesis about the presence of a unit root for a number of first differences is rejected at a 1% significance level.

Model 2: Extended Dickey-Fuller test for the first differences of the dependent variable

Extended Dickey-Fuller test for d_Investment
test. starting from 12 lags, AIC criterion
sample size 100
unit root null hypothesis: \( a = 1 \)

**Constant test**

Figure 1. Dynamics of changes in investments in the period 1993-2020 years
including 10 lag(s) for (1-L)d_Investment
model: (1-L)y = b0 + (a-1)*y(-1) + ... + e
score for (a - 1): -1.09273
test statistic: tau_c(1) = -3.46683
asympt. p-value 0.008908
coefficient 1st order autocorrelations for e: 0.007
difference lag: F(10, 88) = 937.312 [0.0000]

**With constant and trend**
including 10 lag(s) for (1-L)d_Investment
model: (1-L)y = b0 + b1*t + (a-1)*y(-1) + ... + e
estimate for (a - 1): -2.30051
test statistic: tau_c(1) = -4.45681
asympt. p-value 0.001719
coefficient 1st order autocorrelations for e: -0.006
difference lag: F(10, 87) = 1011.842 [0.0000]

The correlogram confirms that the model under consideration has a 4th order autoregression (the corresponding lag is significant). Figure 2 clearly shows seasonality. Let us estimate the parameters of the ARIMA(4,1,0) model for the Investment series.

![Figure 2. Correlogram for the first differences of the time series](image)

The results of model estimation are contained in Tables 1, 2.


<table>
<thead>
<tr>
<th>value</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>const</td>
<td>1.54163e+06</td>
<td>597603</td>
<td>2.580</td>
<td>0.0099</td>
</tr>
<tr>
<td>Phi_1</td>
<td>1.92907</td>
<td>0.104758</td>
<td>18.41</td>
<td>1.00e-075</td>
</tr>
<tr>
<td>Phi_2</td>
<td>-1.56938</td>
<td>0.207035</td>
<td>-7.580</td>
<td>3.45e-014</td>
</tr>
<tr>
<td>Phi_3</td>
<td>1.15053</td>
<td>0.229460</td>
<td>5.014</td>
<td>5.33e-07</td>
</tr>
<tr>
<td>Phi_4</td>
<td>-0.524425</td>
<td>0.133285</td>
<td>-3.945</td>
<td>8.33e-05</td>
</tr>
</tbody>
</table>
Table 2. Model3: ARIMA, dispersion statistics

<table>
<thead>
<tr>
<th></th>
<th>Average hang change</th>
<th>Dependent variable standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-centered R-square</td>
<td>0.990168</td>
<td>0.989895</td>
</tr>
<tr>
<td>Log. credibility</td>
<td>−1472.262</td>
<td>2963.141</td>
</tr>
<tr>
<td>Criterion of Schwartz</td>
<td>2972.835</td>
<td>−58.47622</td>
</tr>
</tbody>
</table>

Thus, in order to predict the dynamics of investments, it is necessary to take into account lag variables, which will be taken into account in the model being developed.

Consider a non-linear model of changes in investments of the current period from the index of gross domestic product and investments of the previous period. A similar model is analyzed in [5].

\[
I_t = A \times I_{t-1} \times \Delta Q_{t-1}
\]

(1)

Where

A – the nature of the technology, linking the level of investment with the dynamics of GDP;

\(I_{t-1}\) – investments of the previous period;

\(\Delta Q_{t-1}\) – is the GDP change index of the previous period.

Since the model includes a lag delay, a partial adjustment method can be used, which allows the most effective leveling of autocorrelation and reflecting the delay in the model.

We take the logarithm of this dependence and construct a linear regression equation:

\[
\text{Ln}I_t = \text{Ln}A + \text{Ln}I_{t-1} + \text{Ln}\Delta Q_{t-1}
\]

(2)

Let us introduce the notation:

\(\text{Ln}I_t = Y_t; \)

\(\text{Ln} A = a_0; \)

\(\text{Ln} I_{t-1} = Y_{t-1}; \)

\(\text{Ln}\Delta Q_{t-1} = X_{t-1}^{1}\)

And get the following linear multiple regression equation:

\[
Y_t = a_0 + Y_{t-1} + X_{t-1} + u
\]

(3)

It should be noted that the values of the factors may contain the problem of multicollinearity, since the values of \(\text{Ln} I_t\) and \(\text{Ln} I_{t-1}\) are closely correlated, due to the fact that they characterize observations with a lag of one period.

When an autoregressive model is considered, the condition that there is no connection between the resulting indicator and the random component is not met. Therefore, it is impossible to use the ordinary least squares method, since its application leads to biased estimates of the estimated parameter [6]. To eliminate multicollinearity, we can use the partial adjustment mechanism, which assumes that our model examines not the actual value of the variable, but its target level.

Let us consider successively the dependence of investments of the current period on the level of investments of the previous period and the dependence of investments of the current period on changes in the real gross domestic product of the previous period. As Wojciech W. Charemza writes: ‘The general autoregressive model is usually described in the form of an autoregressive distributed lag (ADL). This means that the dependent variable is an exponential function of the eigenvalues, as well as the current and lagging values of all independent variables” [7, p.59].

Direct notation of the ADL model uses the lag operator \(L^r\), defined for the variable \(X\) as:

\[
L^r X_t = X_{t-r}
\]

(4)
Scalar polynomials in L are also convenient:

$$\alpha(L) = \sum_{i=m}^{n} \alpha_i L^i$$

so:

$$\alpha(L) X_t = \sum_{i=m}^{n} \alpha_i L^i X_t = \alpha_m X_{t-m} + \alpha_{m+1} X_{t-m-1} + \ldots + \alpha_n X_{t-n}$$

(6)

Thus, $\alpha(L)X_t$ is the weighted sum of the lag values of the variable $X_t$ above level $n$.

The partial adjustment model assumes that the equation will be described not in terms of actual values, but in terms of describing the target level of the resulting indicator.

$$Y_t^* = \alpha + \beta X_{t-1} + \mu_t$$

(7)

The actual change in the dependent variable ($Y_t - Y_{t-1}$) is proportional to the difference between its target level ($Y_t^*$) and the value of the previous period:

$$Y_t - Y_{t-1} = \lambda (Y_t^* - Y_{t-1}) + v_t, \quad (0 \leq \lambda \leq 1)$$

(8)

where

$v_t$ – random member

From Equation 8 we define $Y_t^*$:

$$Y_t = \lambda Y_t^* - \lambda Y_{t-1} + Y_{t-1} + v_t = \lambda Y_t^* + (1 - \lambda) Y_{t-1} + v_t$$

(9)

Substituting into the Equation 7 Equation 5, we get:

$$Y_t = \alpha \lambda + \beta \lambda X_{t-1} + (1 - \lambda) Y_{t-1} + v_t + \lambda \mu_t$$

(10)

Let us estimate the regression equation according to the specification of Equation 10, which uses the logarithms of the considered indicators. Having estimated the dependence of investments of the current period on changes in the real gross domestic product and the level of investments of the previous period, we obtain the parameters of the behavioral model: $\alpha; \beta; \lambda$.

The coefficient at $Y_{t-1}$ gives the value $(1 - \lambda)$, knowing it, you can determine the coefficient $\lambda$. Further, knowing the coefficient $\lambda$, we determine the coefficients $\alpha$ and $\beta$. The resulting model includes a stochastic explanatory factor $Y_{t-1}$. But this variable is now not correlated with the current value of the total random term, since the random terms of the equation $v_t$ and $\mu_t$ are calculated after $Y_{t-1}$ is determined [8].

Let’s consider how the target level of investment correlates with the actual level. The level of investment is proportional to the change in the level of gross domestic product. Many studies have proven rather high correlation between changes in real gross domestic product and changes in the volume of investment. In particular, one of the studies mentions: “Empirical results confirm that in the long run, private, public investments have a positive effect on real gross domestic product” [9].

As a rule, the planned level of economic growth ($\Delta Q^*$) is laid down by the government and is the so-called target that is being sought.

$$\Delta Q_t^* = \kappa \Delta Q_{t-1}$$

(11)

Usually a positive growth trend is planned. In Kazakhstan, the positive dynamics of growth in the index of the physical volume of gross domestic product is influenced by: population growth and the magnitude of development and expansion of technological innovations. However, in Kazakhstan, since 2016, investment growth is mainly associated with public sector investments, the private sector has significantly reduced investments. The pandemic has had a negative impact on the level of investments. The drop in economic growth in 2020 by 2.6% against 2019 was partially offset in 2021, the GDP grew by 4%. However, this level did not allow to overcome the recession of 2020. The low base of 2020 gives false
information about the positive dynamics in relative terms associated with overcoming the recession of the previous period.

The main impact on the recovery of economic growth in 2021 was provided by measures of state support for investment in residential real estate, which partially offset the decline in gross investment in other sectors of the economy. Also in 2021, the value of Kazakhstani exports increased due to a rise in oil production by 6%. The target of economic growth in Kazakhstan for 2022 is economic growth of at least 4%. At the same time, the share of investments in fixed capital in relation to the gross domestic product should be at least 18.5%.

**Model Results**

To test the hypothesis that investments depend on changes in real gross domestic product and investment volumes of the previous period, we used official statistics for the period 1993-2020 on a quarterly basis. Using data series tested for stationarity, we obtained a multiple regression equation that reflects the dependence of investment on changes in real gross domestic product and investment volumes of the previous period. The results of the model are shown in Table 3.

### Model 4: MLS, observations used 1994:2-2020:4 (T = 107)

<table>
<thead>
<tr>
<th>Model of the relationship between investments of the current period from investments of the previous period and the GDP index.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent variable: ( Y_t ).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>value</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda )</td>
<td>3.43600</td>
<td>0.9454932</td>
<td>3.659</td>
<td>0.0078</td>
</tr>
<tr>
<td>( Y_{t-1} )</td>
<td>-2.77280</td>
<td>0.954522</td>
<td>-2.905</td>
<td>0.0045</td>
</tr>
<tr>
<td>( v_t )</td>
<td>-0.300552</td>
<td>0.0468903</td>
<td>-6.410</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>( X_{t-1} )</td>
<td>0.616796</td>
<td>0.0474933</td>
<td>12.99</td>
<td>&lt;0.0001</td>
</tr>
</tbody>
</table>

\[
Y_t = 3.436 + 0.616X_{t-1} - 2.772Y_{t-1} \tag{12}
\]

The regression and dispersion statistics of the resulting equation are shown in Table 4.

### Model 4: MLS. Regression variance statistics of a nonlinear model of the dependence of investments of the current period on investments of the previous period and the GDP index.

<table>
<thead>
<tr>
<th>Dependent variable mean</th>
<th>0.048223</th>
<th>Dependent variable Std. deviation</th>
<th>0.486836</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sum of Sq. remainders</td>
<td>3.051206</td>
<td>Model Std. Error</td>
<td>0.173810</td>
</tr>
<tr>
<td>Non-centered R-square</td>
<td>0.879740</td>
<td>Centered R-square</td>
<td>0.878549</td>
</tr>
<tr>
<td>F(6, 101)</td>
<td>123.1414</td>
<td>P-value (F)</td>
<td>3.70e-44</td>
</tr>
<tr>
<td>Log. credibility</td>
<td>38.48870</td>
<td>Criterion of Akaite</td>
<td>-64.9739</td>
</tr>
<tr>
<td>Parameter rho</td>
<td>-0.011828</td>
<td>h- Darbin statistic</td>
<td>-0.139489</td>
</tr>
</tbody>
</table>

The data in Table 4 allow us to conclude that the relationship between the selected factor characteristics and the effective one is quite strong.

Testing the hypothesis about the absence of dependence of investments of the current period on the GDP index and investments of the previous period gave the following results.

\[
t = \frac{b - \beta_0}{c.o.(b)} = \frac{b - 0}{c.o.(b)} = \frac{0.616}{0.047} = 12.99 \tag{13}
\]

\[
t = \frac{\alpha - \alpha_0}{c.o.(\alpha)} = \frac{\alpha - 0}{c.o.(\alpha)} = \frac{3.436}{0.945} = 3.659 \tag{14}
\]
Comparing the calculated value of \( t \)-statistics with the tabular value, we reject the null hypothesis, concluding that the value of \( \beta \) is actually different from zero, respectively, investments of the current period depend on the change in the GDP index and investments of the previous period.

Assessing the quality of the model obtained through the Fisher coefficient, we can state the following \( F \)-calculated, equal to 123.14, significantly higher than \( F \)-critical, equal to 3.39, respectively, the value of the coefficient of determination did not appear by chance. A greater number of factors influencing the studied indicator is included in the studied model.

Based on the obtained coefficients, the following parameters are determined: \( a; \beta; \lambda \).

\[
1-\lambda = -2.77 \quad \lambda = 1 + 2.77 = 3.77 \\
a\lambda = 3.436 \quad a = 3.436 / 3.77 = 14.559 \\
\beta\lambda = 0.616 \quad \beta = 0.616 / 3.77 = 0.163
\]

The obtained parameters allow us to give an economic interpretation of the results:

In the absence of the influence of the real gross domestic product index and the level of investments of the previous period, autonomous investments are at the level of: \( e^{3.436} = 31.0625 \) million tenge per year, which is an extremely low indicator for economic growth and, accordingly, the economy needs a significant infusion of direct investment.

Autonomous investments do not correlate with changes in the country’s national income, they serve to restore capital and upgrade it [10]. In a period of rapidly changing technological conditions and, accordingly, changes in the tastes and preferences of consumers, it is autonomous investments that make it possible to produce products of higher quality and meeting modern requirements. Also in high-tech industries, it is autonomous investments that make it possible to lead the industry by increasing the productivity of factors, both capital and labor.

The degree of sensitivity of the investments of the current period to the investments of the previous period is equal to \( e^{0.616} = 1.8517 \). With an increase in investments of the previous period by 1 million tenge, investments in the current period will increase by 1.8517 million tenge. Thus, the level of investment in the current period will have a double effect on the level of investment in the next period, which leads to a multiplier effect of investment growth. On the other hand, if in the current period the level of investment is insufficient, then this will also have a multiplicative effect on the level of investment in the next period and a decrease in economic growth.

The change in investments of the current period from the rate of change in real gross domestic product will be: \( e^{-2.77} = 0.0627 \). The growth of real gross domestic product by 1 million tenge leads to an increase in investments by 0.0627 million tenge. Thus, we see that Kazakhstan has a rather low level of capital intensity ratio, which characterizes the declining trend of induced investment and indicates the importance of increasing economic growth rates for the manifestation of a multiplier growth of induced investment.

The impact of the GDP index is lower than the impact of the investment level of the previous period. Since the dynamics of the GDP index has decreased in the last few years, this is directly reflected in the decline in the dynamics of induced investment. The average growth of the index of the physical volume of GDP in the period 2000-2006 was 10.3%, it corresponded to the index of the physical volume of investments equal to 26%. In the period 2007-2020, the growth rate of the GDP dynamics decreased to 3.9%, which was reflected in the decrease in the index of the physical volume of investments to the level of 10.3% [11].

Conclusion

In conclusion, it should be noted that the expected economic growth, measured through the growth of the real gross domestic product index, requires an increase in investment in the current period. According to the accelerator principle, the proportionality coefficient between the volume of investment and the change in gross domestic product is greater than 1, so the growth of real output requires an increase in investment spending.

Thus, for sustainable economic growth during economic downturns caused by both economic and other reasons, it is necessary to compensate for the decline in investment in the current period, in order to reduce the multiplier effect of depressed investment in the future period and, as a consequence, the slowdown in economic growth.
The current economic situation in the world shows the importance of investment in the real sector of the economy of any country.

Specialist who are dealing with investments into securities describe that in primitive economies the bulk of investments are real investments, while in the modern economy most of the investments are represented by financial investments. Nowadays the statement above is being refuted by current economic situation in the world. [12, c.1]. The importance of investments in the real sector of the economy is confirmed by the unstable state of the economies of the developed countries of the world.

References:
11 Key socio-economic indicators. [Electronic resource]- URL: https://stat.gov.kz