

МАТЕМАТИКА ЖӘНЕ ФИЗИКАЛЫҚ ПРОЦЕСТЕР МЕН МЕХАНИКАЛЫҚ ЖҮЙЕЛЕРДІ МАТЕМАТИКАЛЫҚ МОДЕЛЬДЕУ

МАТЕМАТИКА И МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ ФИЗИЧЕСКИХ ПРОЦЕССОВ И МЕХАНИЧЕСКИХ СИСТЕМ

MATHEMATICS AND MATHEMATICAL MODELING OF PHYSICAL PROCESSES AND MECHANICAL SYSTEMS

IRSTI 28.23.29, 28.17.23

<https://doi.org/10.51889/2959-5894.2026.95.3.001>

B.Zh. Abdullayeva¹, A.R. Eralieva², T.E. Amanzholov^{2*}, M.S. Tungatarova¹, D.Y. Aizhulov¹

¹ K.I. Satpayev Kazakh National Research Technical University, Almaty, Kazakhstan¹

² Al-Farabi Kazakh National University, Almaty, Kazakhstan²

*e-mail: tannur.amanzholov@gmail.com

SENSITIVITY ANALYSIS OF DESIGN AND OPERATING PARAMETERS AFFECTING FLAT-PLATE SOLAR COLLECTOR EFFICIENCY IN BTES

Abstract

Flat-plate solar collectors are crucial in Borehole Thermal Energy Storage systems for bridging the gap between renewable energy supply and consumer heat demand. Because most studies rely on idealized models, this work develops a Python-based transient mathematical model of a "solar collector–heat exchanger" system using energy balance equations and real commercial SchücoSol collector specifications. With this verified model, a parametric and sensitivity analysis ($\pm 20\%$) evaluated how daily efficiency is impacted by optical efficiency, heat losses, heat exchanger transfer coefficient, fluid mass flow rate, and ambient temperature. Results demonstrate that ambient temperature, optical efficiency, linear heat losses, and heat exchanger characteristics most significantly influence efficiency. These findings quantitatively assess the impact of design and operating parameters, offering practical recommendations for designing and optimizing real seasonal thermal energy storage systems.

Keywords: solar collector, borehole thermal energy storage (BTES), seasonal thermal energy storage (STES), heat exchanger, modeling, optical efficiency.

Б.Ж. Абдуллаева¹, А.Р. Ералиева², Т.Е. Аманжолов², М.С. Тунгатарова¹, Д.Е. Айжулов¹

¹Казахский национальный исследовательский технический университет имени К. И. Сатпаева,
г.Алматы, Казахстан

²Казахский национальный университет имени аль-Фараби, г.Алматы, Казахстан

АНАЛИЗ ЧУВСТВИТЕЛЬНОСТИ ЭФФЕКТИВНОСТИ ПЛОСКОГО СОЛНЕЧНОГО КОЛЛЕКТОРА В ГТА К КОНСТРУКТИВНЫМ И ЭКСПЛУАТАЦИОННЫМ ПАРАМЕТРАМ

Аннотация

Плоский солнечный коллектор является ключевым элементом систем грунтового теплового аккумулирования, необходимых для компенсации дисбаланса между потребительским спросом на тепловую энергию для нужд отопления и ГВС и предложением энергии из возобновляемых источников. Выполняя сбор солнечной энергии, коллектор значительно влияет на общую эффективность всей системы. Поскольку большинство исследований опирается на идеализированные модели, в данной работе разработана нестационарная математическая модель системы «солнечный коллектор – теплообменник» на базе уравнений баланса энергии. Модель реализована в Python с

использованием характеристик реального промышленного коллектора SchücoSol. На основе верифицированной модели выполнены параметрическое исследование и анализ чувствительности ($\pm 20\%$) влияния оптического КПД, тепловых потерь, коэффициента теплопередачи теплообменника, расхода теплоносителя и температуры среды на суточную эффективность. Установлено, что наибольшее влияние на КПД оказывают температура среды, оптический КПД, линейные тепловые потери и характеристики теплообменника. Результаты позволяют количественно оценить влияние конструктивных и режимных параметров и сформулировать рекомендации по проектированию и оптимизации реальных систем сезонного аккумулирования энергии.

Ключевые слова: солнечный коллектор, грунтовый тепловой аккумулятор (ГТА), сезонное хранение тепловой энергии, теплообменник, моделирование, оптический КПД

Б.Ж. Абдуллаева¹, А.Р. Ералиева², Т.Е. Аманжолов², М.С. Тунгатарова¹, Д.Е. Айжулов¹

¹Қ.И. Сәтбаев атындағы Қазақ ұлттық зерттеу техникалық университеті, Алматы қ., Қазақстан

²Әл-Фараби атындағы Қазақ ұлттық университеті, Алматы қ., Қазақстан

ЖЕР АСТЫ ЖЫЛУ САҚТАУ ЖҮЙЕСІНДЕГІ ЖАЗЫҚ КҮН КОЛЛЕКТОРЫНЫҢ ТИІМДІЛІГІНІҢ КОНСТРУКЦИЯЛЫҚ, ПАЙДАЛАНУ ПАРАМЕТРЛЕРІНЕ СЕЗІМТАЛДЫҒЫН ТАЛДАУ

Аңдатпа

Жазық күн коллекторы – жаңартылатын энергия мен тұтынушылық сұраныс арасындағы теңгерімсіздікті жоюға арналған жерасты жылу сақтау жүйелерінің негізгі элементі. Күн энергиясын жинақтай отырып, ол бүкіл жүйенің тиімділігіне айтарлықтай әсер етеді. Көптеген зерттеулер идеалдандырылған модельдерге сүйенетіндіктен, бұл жұмыста «күн коллекторы – жылу алмастырғыш» жүйесінің стационарлық емес математикалық моделі әзірленді. Модель нақты өнеркәсіптік SchücoSol коллекторының сипаттамалары қолданылып, Python тілінде жүзеге асырылды. Верификацияланған модель негізінде оптикалық ПӘК, жылу шығындары, жылу алмастырғыштың жылу беру коэффициенті, жылу тасымалдағыш шығыны және қоршаған орта температурасының тәуліктік тиімділікке әсерін бағалау үшін параметрлік және сезімталдық ($\pm 20\%$) талдауы жүргізілді. Нәтижесінде ПӘК-ке қоршаған орта температурасы, оптикалық ПӘК, сызықтық жылу шығындары және жылу алмастырғыш сипаттамалары басты әсер ететіні анықталды. Алынған мәліметтер құрылымдық және режимдік параметрлердің әсерін сандық бағалауға, сондай-ақ маусымдық энергия жинақтаудың нақты жүйелерін жобалау мен оңтайландыру бойынша практикалық ұсыныстар әзірлеуге мүмкіндік береді.

Түйін сөздер: күн коллекторы, жер асты жылу аккумуляторы, жылу энергиясын маусымдық жинақтау, жылу алмастырғыш, модельдеу, оптикалық ПӘК.

Introduction

Heating and cooling account for approximately half of Europe's energy consumption, with 85% supplied by fossil fuels [1], while buildings consume up to 40% of the EU's total energy [2]. In Canada, space and water heating generate over 80% of residential greenhouse gas emissions [3]. These figures highlight the need for low-carbon heating systems. Solar energy offers a promising alternative, but its availability is seasonally mismatched with heating demand: solar radiation peaks in summer, whereas heat demand peaks in winter [4]. Seasonal Thermal Energy Storage (STES) addresses this imbalance by storing surplus renewable heat collected in summer for winter use [5].

Among underground sensible heat storage technologies, Borehole Thermal Energy Storage (BTES) offers flexibility, scalability, and cost-effectiveness [6]. It uses soil or rock as the storage medium (Figure 1), exchanging heat through arrays of vertical boreholes, typically 30–200 m deep, equipped with U-shaped tubes carrying a heat-transfer fluid [6, 7]. Unlike tank storage or Aquifer Thermal Energy Storage (ATES), BTES requires no artificial reservoirs and is independent of aquifer availability. Its modular design allows expansion by drilling additional boreholes [7].

Integrating solar collectors with seasonal storage forms the basis of Central Solar Heating Plants with Seasonal Storage [2]. Collectors supply surplus summer heat, which a circulating fluid transfers through heat exchangers to BTES for storage until winter. A notable example is Canada's Drake Landing Solar Community (DLSC), completed in 2006. It combines 800 flat-plate collectors installed

on garage roofs with 144 boreholes, each 35 m deep [8]. After five years of operation, DLSC achieved a solar fraction of 97%, meeting nearly all the heating needs of 52 homes with stored summer solar energy [6]. Collector selection strongly influences overall STES performance. Flat-plate solar collectors (FPCs) are widely used because of their reliability and suitability for low-grade heat [9]. However, their efficiency decreases substantially at fluid temperatures of 70–95°C, often required for effective seasonal storage charging [10].

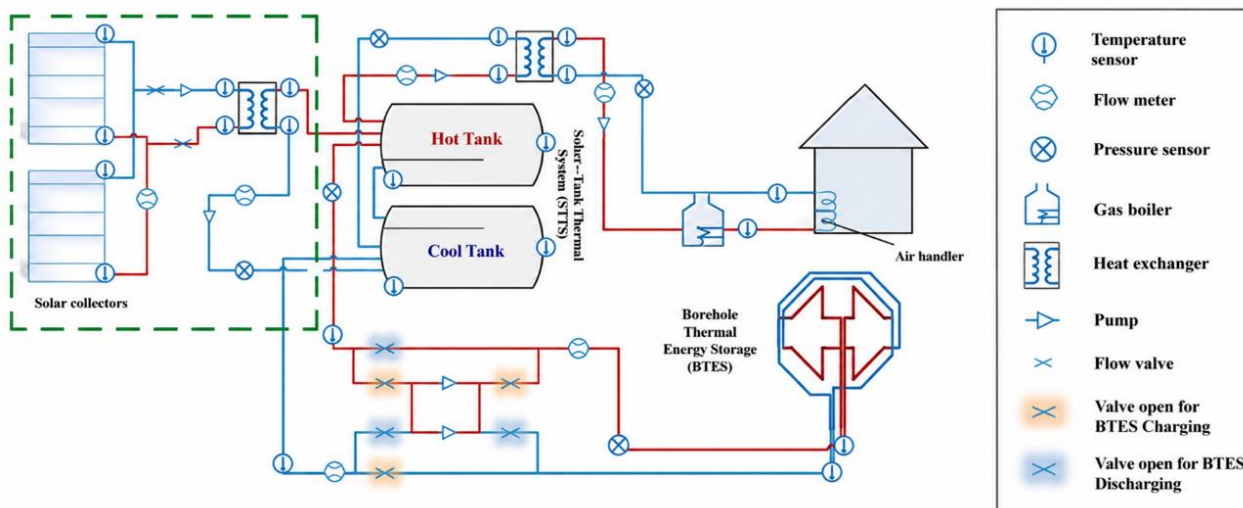


Figure 1. Schematic diagram of a borehole thermal energy storage system

Hybrid configurations improve solar heat collection at high temperatures. At the Taars plant in Denmark, flat-plate collectors preheat the fluid, while parabolic trough collectors (PTCs) raise its temperature to approximately 95°C for storage or district heating [10].

Solar–STES integration nevertheless faces several challenges. As the borehole field warms, higher collector inlet temperatures reduce collection efficiency, while high storage temperatures increase ground heat losses. At DLSC, approximately 60% of the injected heat was lost [2]. Insufficient summer replenishment also causes progressive ground cooling, reducing heat pump performance and increasing electricity consumption [11]. Furthermore, drilling and collector installation costs hinder commercial deployment without substantial subsidies [6]. Low-temperature BTES coupled with ground-source heat pumps offers a potential solution. Storage temperatures below approximately 35°C reduce heat losses and improve collector efficiency; heat pumps then raise the temperature for building heating [6]. Phase-change materials can increase storage density and moderate temperature fluctuations [12], while hybrid FPC–PTC systems improve heat generation across different temperature ranges [10]. However, many simulations inadequately represent practical operation. Electricity-price-based control of heat pumps and storage charging remains insufficiently explored [11]. Balancing collector array size, borehole storage costs, and thermal performance to minimize the levelized cost of heat is another unresolved challenge [10, 13]. Detailed analysis using commercial collector specifications is therefore needed to assess how actual design characteristics affect seasonal efficiency. This study investigates the sensitivity of collector thermal efficiency to design and operating parameters. The objectives are to compare model results with published calculations [14], use commercial collector specifications, and assess subsystem performance relevant to STES integration. Ground storage dynamics and seasonal BTES efficiency are outside the present model.

Research methodology

Thermal state model of a solar collector. The modeled subsystem comprises a flat-plate collector, a primary circulation loop, and a heat exchanger within the solar circuit shown in Figure 1. It supplies heat to the downstream storage system; the borehole field itself is not modeled. The collector comprises an insulated housing, a transparent cover, an absorber, and copper tubes (Figure 2a). Solar

radiation absorbed by the plate is transferred through the tubes to the circulating heat-transfer fluid, while conduction, convection, and radiation cause heat losses (Figure 2b).

The heat exchanger contains primary and secondary sections separated by a heat-transfer partition (Figure 2c). The primary fluid releases heat received from the collector, while the secondary fluid transports it to the storage tank.

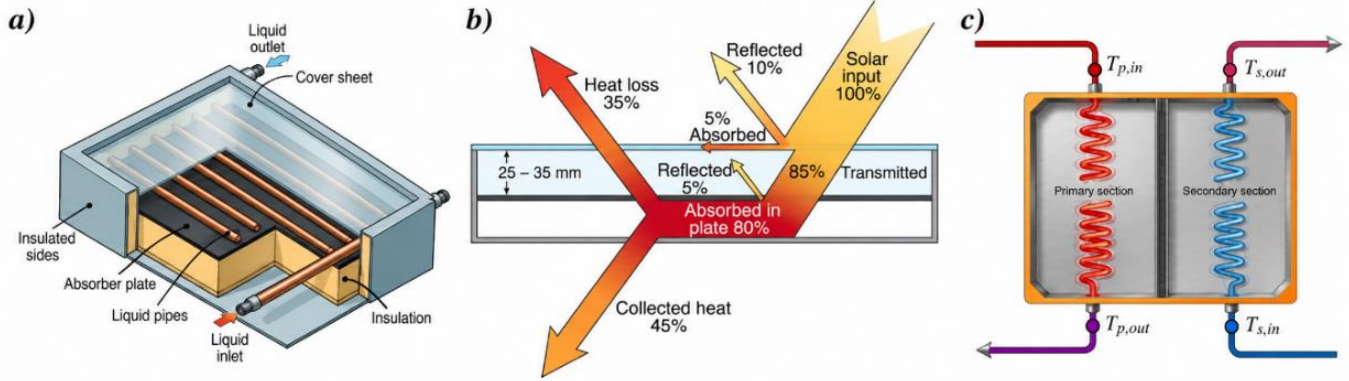


Figure 2. Solar thermal system components: (a) flat-plate solar collector construction; (b) solar energy distribution and heat losses in the collector; (c) heat exchanger schematic showing the primary and secondary sections and fluid inlet and outlet temperatures.

The model predicts the time-dependent collector outlet temperature and the heat exchanger outlet temperatures in both circuits, T_1 and T_2 , for a prescribed daily solar radiation profile. The following assumptions apply:

- 1) Each modeled fluid volume is perfectly mixed, with uniform temperature and no stratification.
- 2) Fluid thermal properties and flow rates in both circuits remain constant.
- 3) The absorber temperature equals approximately the mean collector inlet and outlet fluid temperature.
- 4) Connecting pipelines are perfectly insulated; their heat losses are neglected.

The collector energy balance includes absorbed solar power, thermal accumulation, environmental losses, and heat removal by the circulating fluid:

$$\dot{Q}_{st} = \dot{Q}_{in} - \dot{Q}_{out} + \dot{Q}_g \quad (1)$$

where $\dot{Q}_{st}$ is the thermal accumulation rate, $\dot{Q}_{in}$ is absorbed solar power, $\dot{Q}_{out}$ comprises environmental losses and fluid heat removal, and $\dot{Q}_g$ is auxiliary heating power (all in W). Here, $\dot{Q}_g = 0$.

Thermal accumulation in the collector fluid is expressed as:

$$\dot{Q}_{st} = \rho c \frac{dT_{c,out}}{dt} V \quad (2)$$

where ρ [kg/m³] is fluid density, c_p [J/(kg·K)] is specific heat capacity, V [m³] is fluid volume, and $T_{c,out}$ [°C] is collector outlet temperature.

The solar energy arriving at the collector surface is determined by expression (3).

$$\dot{Q}_{sol} = IA \quad (3)$$

The absorbed solar power accounts for cover transmittance and absorber absorptance:

$$\dot{Q}_{in} = \dot{Q}_{abs} = I\eta_0 A \quad (4)$$

where $I [W \cdot m^{-2}]$ – solar radiation falling on the collector surface, $A [m^2]$ – collector area, $\eta_0 = \tau\alpha$ – optical efficiency of the collector, τ – transmittance of a transparent coating, α – absorption coefficient of the absorber.

The total heat leaving the collector control volume comprises environmental losses and useful heat transferred to the fluid:

$$\dot{Q}_{out} = \dot{Q}_{loss} + \dot{Q}_F \quad (5)$$

where $\dot{Q}_{loss} [W]$, the heat losses of the collector to the environment are described as:

$$\dot{Q}_{loss} = U_L A (T_{abs} - T_{amb}) \quad (6)$$

where $U_L [W \cdot m^{-2} \cdot ^\circ C^{-1}]$ – heat loss coefficient, $T_{abs} [^\circ C]$ – absorber temperature, $T_{amb} [^\circ C]$ – ambient temperature.

The thermal power $\dot{Q}_F [W]$ transferred to the coolant is determined by the expression:

$$\dot{Q}_F = \dot{m} c (T_{c,out} - T_{c,in}) \quad (7)$$

where $\dot{m} [kg/s]$ is mass flow rate, $c [J/(kg \cdot K)]$ is fluid specific heat capacity, and $T_{c,in} [^\circ C]$ is collector inlet temperature.

Based on the systems of equations (1)–(7), a differential equation for the heat balance of the collector is compiled:

$$C \frac{dT_{c,out}}{dt} = I A \eta_0 - U_L A (T_{abs} - T_{amb}) - \dot{m} c (T_{c,out} - T_{c,in}) \quad (8)$$

where $C = \rho c p V [J/K]$ is the fluid heat capacity and $t [s]$ is time.

Equation (8) describes coolant temperature evolution under solar input, environmental losses, and fluid flow. Only dominant upper-surface heat losses are considered. Coolant density, specific heat capacity, and volume are assumed constant [6], while the absorber temperature equals the mean coolant temperature:

$$T_{abs} = T_{avg} = \frac{T_{c,in} + T_{c,out}}{2} \quad (9)$$

where $T_{abs} [^\circ C]$ – average temperature of the coolant in the collector. Substituting (9) into (8) yields the governing equation for the flat-plate solar collector:

$$\frac{dT_{c,out}}{dt} = \frac{I A \eta_0}{C} - \frac{U_L A}{C} (T_{avg} - T_{amb}) - \frac{\rho F c}{C} (T_{c,out} - T_{c,in}) \quad (10)$$

where $F_c [m^3/s]$ is the primary volumetric flow rate.

The heat exchanger is represented by two interacting, perfectly mixed fluid volumes, coupled through thermal conductance UA . The primary outlet temperature satisfies:

$$\frac{dT_{p,out}}{dt} = \frac{F_c}{V_1} (T_{p,in} - T_{p,out}) - \frac{UA}{\rho c V_1} (T_{p,out} - T_{s,out}) \quad (11)$$

where $T_{p,in}$ and $T_{p,out}$ are primary inlet and outlet temperatures, $T_{s,out}$ is secondary outlet temperature, V_1 is primary fluid volume, and $UA [W/K]$ is heat exchanger conductance. With insulated connections, $T_{p,in} = T_{c,out}$ and $T_{p,out} = T_{c,in}$.

The change in the coolant temperature at the outlet of the secondary circuit is determined by the expression:

$$\frac{dT_{s,out}}{dt} = \frac{F_1}{V_2} (T_{s,in} - T_{s,out}) + \frac{UA}{\rho c V_2} (T_{p,out} - T_{s,out}) \quad (12)$$

where V_2 is secondary fluid volume, $T_{s,in}$ is secondary inlet temperature, and F_1 is secondary volumetric flow rate [m^3/s]. A variable solar insolation profile was used to simulate the collector's daily operating mode. The daily variation in solar radiation was approximated by a sinusoidal function between sunrise and sunset:

$$I(t) = I_{\max} \sin \left[\frac{\pi(t-t_{\text{rise}})}{t_{\text{set}}-t_{\text{rise}}} \right] \quad (13)$$

where I [$W \cdot m^{-2}$] – intensity of solar radiation at a given moment in time t , $I_{\max}$ [$W \cdot m^{-2}$] – the maximum intensity of solar radiation corresponding to a clear summer day ($I_{\max} = 900 W \cdot m^{-2}$), t_{rise} – sunrise time, t_{set} – sunset time. The instantaneous efficiency of the collector was determined by the formula:

$$\eta = \eta_0 - k_1 \frac{T_{\text{avg}} - T_{\text{amb}}}{I} - k_2 \frac{(T_{\text{avg}} - T_{\text{amb}})|T_{\text{avg}} - T_{\text{amb}}|}{I} \quad (14)$$

where η – instantaneous collector efficiency, k_1 [$W \cdot m^{-2} \cdot ^\circ C^{-1}$] – linear heat loss coefficient, k_2 [$W \cdot m^{-2} \cdot ^\circ C^{-2}$] – coefficient of square heat loss. The daily efficiency of the collector was estimated by the integral efficiency:

$$\eta_{\text{day}} = \frac{\int (Q_{\text{sol}} - Q_{\text{loss}}) dt}{A \int I dt} \quad (15)$$

where η_{day} – daily efficiency of the system. The initial conditions for the numerical simulation were set based on the assumption that at the initial moment of time the entire system is in thermal equilibrium with the environment:

$$T_{c,out}(0) = T_{c,in}(0) = T_{p,in}(0) = T_{p,out}(0) = T_{s,in}(0) = T_{s,out}(0) = T_a \quad (16)$$

The model was implemented in Python using NumPy. Figure 3 compares collector and heat exchanger temperatures with published calculations [14]. In this verification case, circulation is initially absent ($F_c = 0$), producing a collector temperature peak; pump activation then transfers heat to the exchanger. This comparison checks implementation consistency rather than providing experimental validation. The subsequent daily simulations use SchücoSol K specifications and a separate solar input profile.

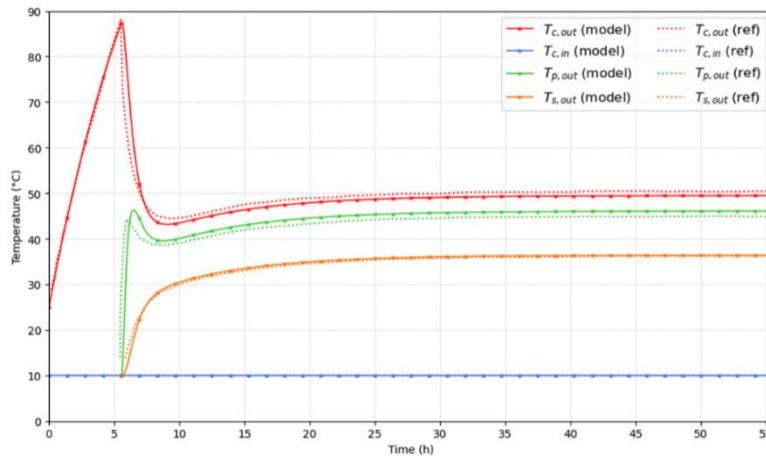


Figure 3. Comparison of the calculated temperature data of the model with the data of work [14]

Discussion of Results

Collector specifications were taken from the SchücoSol K product sheet [15] after comparison with published calculations [14]. Each of six parameters was varied independently while the others remained at their baseline values (Table 1). Daily collector efficiency was the evaluation criterion. The full-range efficiency spread is defined as:

$$\Delta\eta = \eta_{\max} - \eta_{\min} \quad (17)$$

Table 1. Research parameters and variation ranges

Parameter	Base value	Variation range	Unit
Optical efficiency η_0	0,799	0,60 – 0,92	–
Linear losses k_1	3,97	1,5 – 8,0	$W/(m^2 \cdot K)$
Square-law losses k_2	0,016	0,004 – 0,040	$W/(m^2 \cdot K^2)$
Pump flow rate F_c	2,5	0,5 – 8,0	l/min
UA of the heat exchanger	45	10 – 150	W/K
Air temperature T_a	10	–10 – +40	$^{\circ}C$

Figure 4 presents the baseline daily simulation using $\eta_0 = 0.799$, $k_1 = 3.97 \text{ W/(m}^2\cdot\text{K)}$, $k_2 = 0.016 \text{ W/(m}^2\cdot\text{K}^2)$, and $A = 2.32 \text{ m}^2$. Solar collection occurs between 06:00 and 18:00, with the collector outlet reaching approximately $66^{\circ}C$ (Figure 4a). The heat exchanger temperatures remain nearly constant after circulation stops, while the collector cools. Figure 4b shows absorbed solar power, environmental heat losses, and useful heat collection, expressed in W.

Figure 5 shows the influence of six parameters on daily collector efficiency, with a common baseline of 48.5%. Increasing η_0 from 0.60 to 0.92 raises efficiency from 36.9% to 55.4%; increasing T_a from -10 to $40^{\circ}C$ raises it from 41.3% to 58.9%.

UA and F_c show diminishing returns. Efficiency rises from 30.4% at $UA = 10 \text{ W/K}$ to 54.3% at 150 W/K , while increasing flow from 0.5 to 8 L/min raises it from 44.7% to 49.2%. Higher flow therefore offers limited additional thermal benefit.

Increasing k_1 from 1.5 to $8.0 \text{ W/(m}^2\cdot\text{K)}$ reduces efficiency from 58.9% to 37.2%; increasing k_2 from 0.005 to $0.040 \text{ W/(m}^2\cdot\text{K}^2)$ reduces it from 50.5% to 45.3%. The linear loss coefficient has the larger influence over these ranges.

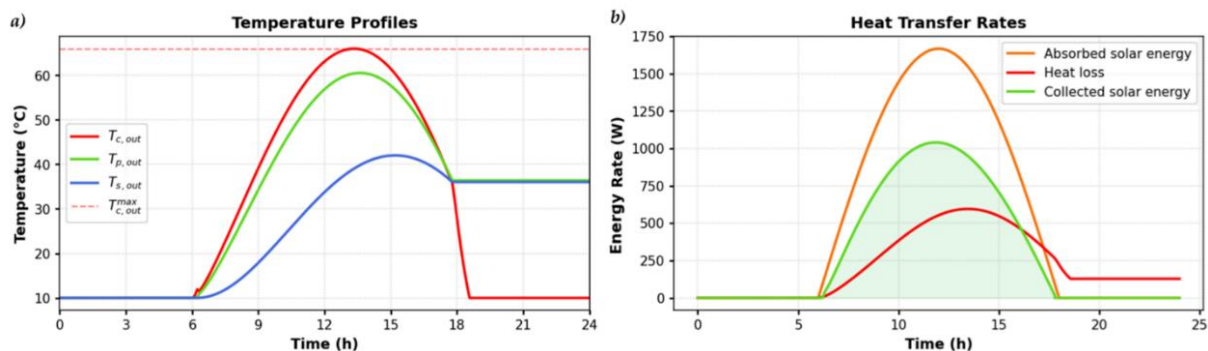


Figure 4. Baseline daily operation: (a) fluid temperatures; (b) absorbed solar power, heat losses, and useful heat collection.

Figure 5 summarizes the influence of the full investigated parameter ranges on daily collector efficiency. The largest efficiency spread is associated with the heat exchanger conductance UA (23.9 percentage points), followed by the linear heat loss coefficient k_1 (21.7), optical efficiency η_0 (18.5), and ambient temperature T_a (17.6). The corresponding spreads for the quadratic heat loss coefficient k_2 and volumetric flow rate F_c are 5.2 and 4.5 percentage points, respectively.

These results characterize the overall influence of the selected parameter ranges and should not be interpreted as normalized local sensitivities.

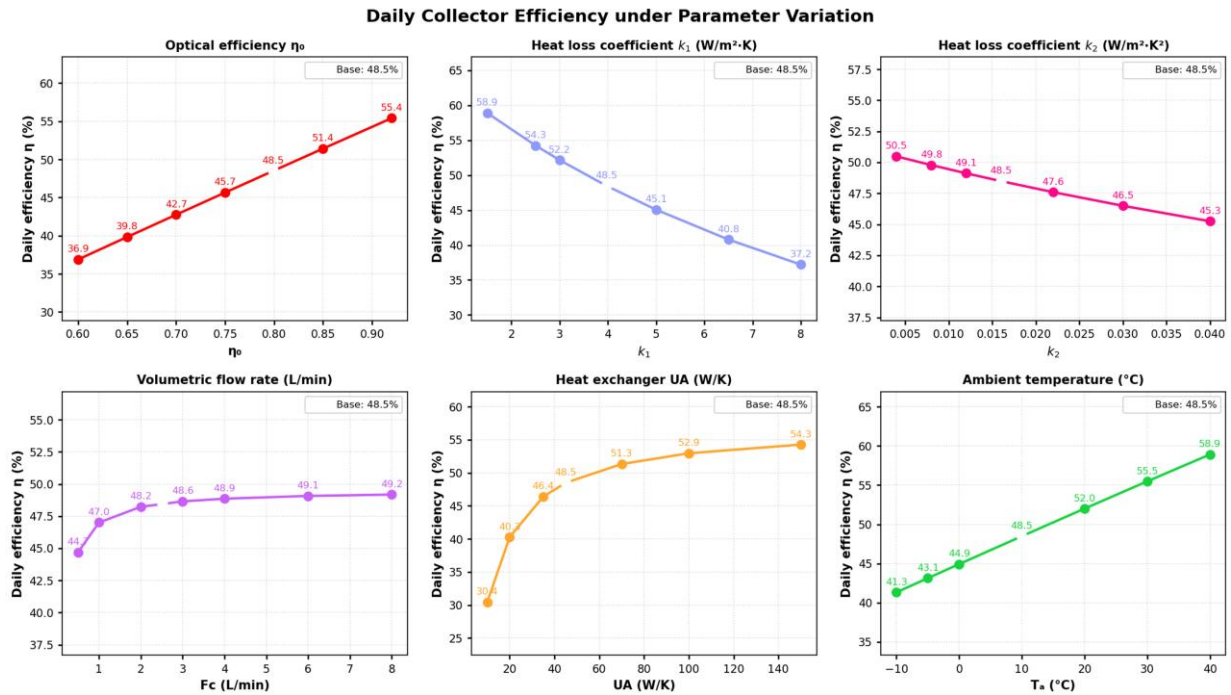


Figure 5. Dependence of the daily efficiency of a solar collector on changes in key operational, design parameters and external factors

Figure 6a presents a local sensitivity analysis of daily collector efficiency, in which each parameter is varied by $\pm 20\%$ around its baseline value. This analysis evaluates the response of collector efficiency to moderate parameter variations under baseline operating conditions and complements the broader parametric analysis presented in Figure 5.

Figure 6b compares the operating trajectory with the instantaneous efficiency characteristic. During high insolation, operating points occupy the low reduced-temperature region, with efficiency approximately 0.55–0.73.

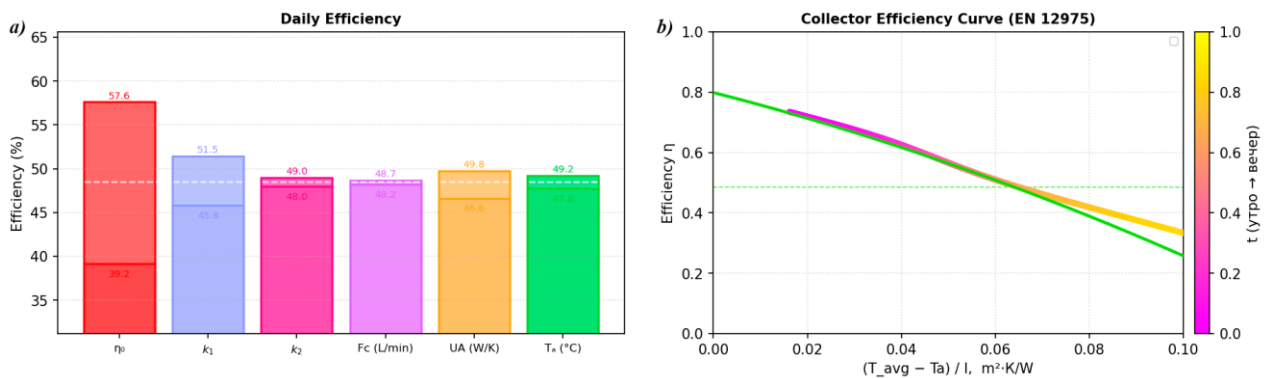


Figure 6. a) Daily efficiency ranges under $\pm 20\%$ variations in the model parameters; (b) instantaneous efficiency characteristic and operating trajectory.

Figure 7 compares useful collected heat and maximum collector outlet temperature. Increasing optical efficiency or ambient temperature increases both quantities, whereas increasing either heat

loss coefficient reduces them. The useful heat gain with higher optical efficiency is accompanied by a higher peak temperature.

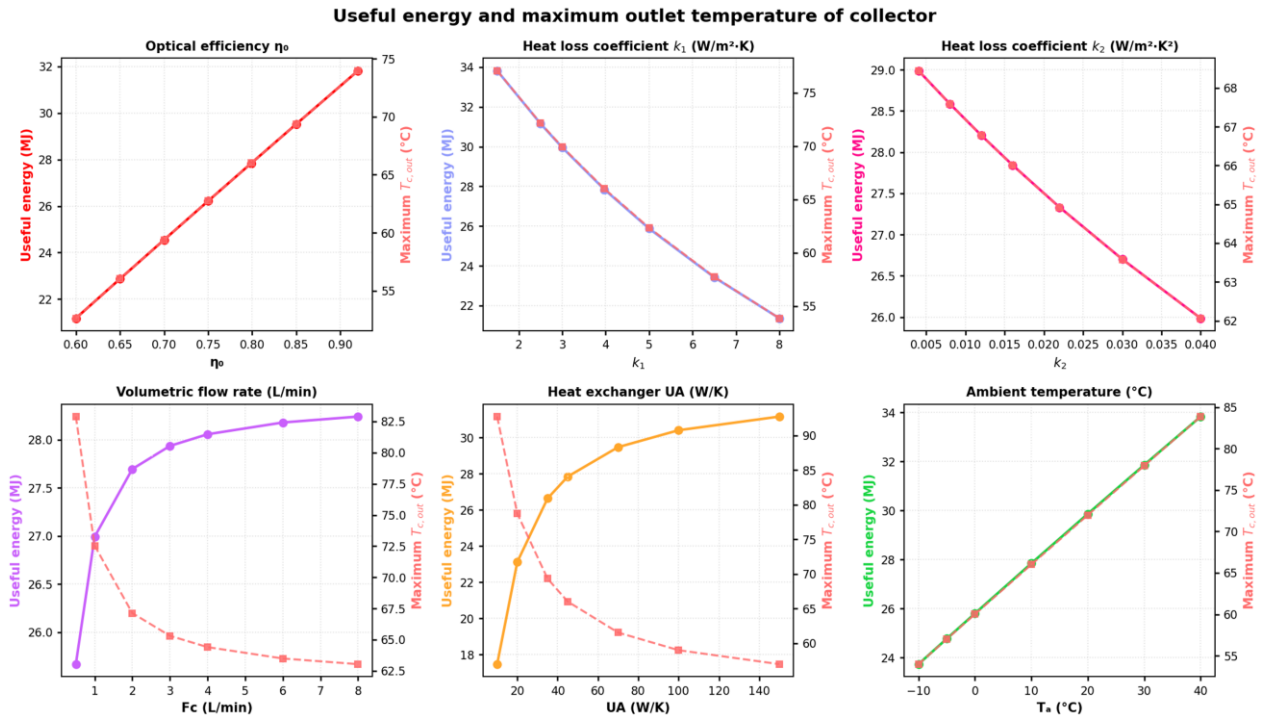


Figure 7. Results of the coupled analysis of useful thermal energy and peak temperature conditions of a solar collector

F_c and UA instead produce opposing energy and temperature trends. Increasing F_c from 0.5 to 8 L/min raises useful heat from approximately 25.7 to 28.2 MJ while reducing the peak temperature from 83 to 63°C. Increasing UA from 10 to 150 W/K raises useful heat from approximately 17.5 to 31 MJ and lowers the peak temperature from 93 to 58°C. Improved heat removal therefore limits overheating, although gains diminish at higher flow and conductance.

Table 2 summarizes the daily efficiency endpoints displayed in Figure 5. Its spreads describe the selected full variation ranges, which differ from the local perturbations in Figure 8. They do not establish a universal sensitivity ranking or an economic optimum.

Table 2. Daily collector efficiency ranges read from Figure 5

Parameter	η_{min} (%)	η_{max} (%)	$\Delta\eta$ (percentage points)
Air temperature T_a	41,3	58,9	17,6
Optical efficiency η_0	36,9	55,4	18,5
Linear heat loss coefficient k_1	37,2	58,9	21,7
Heat exchanger conductance UA	30,4	54,3	23,9
Pump flow rate F_c	44,7	49,2	4,5
Quadratic heat loss coefficient k_2	45,3	50,5	5,2

Figure 8 presents local sensitivity around the 48.5% baseline. Optical efficiency has the largest reported effect, followed by U_{eff} and UA ; T_a and F_c have smaller effects. This five-parameter analysis differs from the six-parameter sweeps in Figures 5–6 because it uses an effective loss coefficient U_{eff} . Local sensitivities describe behavior near the baseline and cannot be ranked directly against the full-range efficiency spreads.

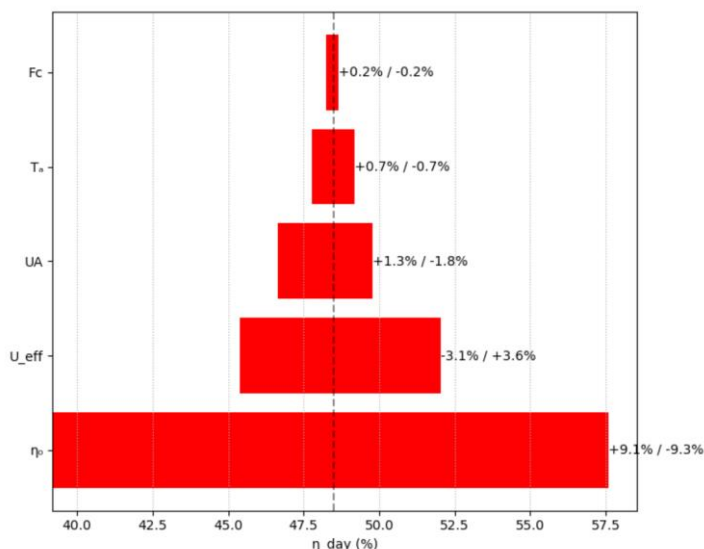


Figure 8. Reported local sensitivity of daily efficiency to $\pm 20\%$ parameter variations

Conclusions

In this paper, the heat transfer processes of a flat-plate solar collector in a water heating system were studied using dynamic mathematical modeling. The following results were obtained and the proposed problems were solved.

A non-stationary mathematical model of the solar collector was developed based on the equations of EN 12975 and implemented numerically using the Euler method with a 10-s step size. The model takes into account the thermal inertia of the system ($C = 19,502 \text{ J/K}$), heat transfer between the elements, and the dynamics of the storage tank. Verification showed good agreement between the instantaneous efficiency and the EN 12975 specification (deviation $\leq 8 \cdot 10^{-4}$). A parametric analysis based on the Hottel–Whillier–Bliss equation revealed the following dynamics of parameter influences on daily system efficiency:

- Across full physical parameter ranges, daily efficiency varied from 30.4% to 58.9%. Under these conditions, the largest efficiency spread ($\Delta\eta$) was observed when varying heat exchanger conductance (UA , up to 23.9 percentage points), linear heat loss coefficient (k_1 , 21.7 percentage points), optical efficiency (η_0 , 18.5 percentage points), and ambient temperature (T_a , 17.6 percentage points). The influence of the quadratic heat loss coefficient (k_2) and coolant flow rate (F_c) remained minor ($\Delta\eta \leq 5.2$ percentage points).

- Local sensitivity analysis ($\pm 20\%$ variation): Optical efficiency (η_0) emerged as the dominant factor, causing an efficiency spread of 18.4 percentage points (ranging between 39.2% and 57.6%). The impact of other parameters under local variations was less pronounced: k_1 yielded a spread of 5.7 percentage points, UA 3.1 percentage points, T_a 1.4 percentage points, k_2 1.0 percentage point, and coolant flow rate F_c only 0.4 percentage points.

Based on these findings, practical design recommendations for cold-climate regions (specifically Kazakhstan) include prioritizing collectors with low linear heat losses ($k_1 \leq 2.5 \text{ W/(m}^2 \text{ K)}$), high optical efficiency ($\eta_0 \geq 0.85$), and sufficient heat exchanger conductance ($UA \geq 50 \text{ W/K}$). Furthermore, operating at moderate coolant flow rates ($F_c \approx 2.5 - 3 \text{ L/min}$) strikes an optimal balance between thermal performance and pumping power requirements. Future research will focus on extending the dynamic model by integrating hybrid energy sources, phase-change materials, and optimizing the thermal capacity of storage systems.

Acknowledgment

This research has been funded by the Committee of Science of the Ministry of Science and Higher Education of the Republic of Kazakhstan (Grant No. AP26103573).

References

- [1] Borko, K. *Insights into Aquifer and Borehole Thermal Energy Storage Systems for Slovenia's Energy Transition* [Text] / K. Borko [et al.] // *Energies*. – 2025. – Vol. 18, no. 5. – P. 1019. – DOI: 10.3390/en18051019.
- [2] Pavlov, G. K. *Seasonal Ground Solar Thermal Energy Storage - Review of Systems and Applications* [Text] / G. K. Pavlov, B. W. Olesen // *Proceedings*. – 2011.
- [3] Mesquita, L. *Drake Landing Solar Community: 10 Years of Operation* [Text] / L. Mesquita [et al.] // *Proc. ISES Solar World Congress 2017 / IEA SHC International Conference on Solar Heating and Cooling for Buildings and Industry*. – Abu Dhabi, UAE, 2017. – DOI: 10.18086/swc.2017.06.09.
- [4] Kamali, M. *Solar-Assisted Seasonal Aquifer Thermal Energy Storage in a Relatively Deep Geothermal Aquifer for Urban Heating: A Canadian Case Study* [Text] / M. Kamali [et al.] // *Processes*. – 2026. – Vol. 14, no. 10. – P. 1636. – DOI: 10.3390/pr14101636.
- [5] Ghoreishi-Madiseh, S. A. *Application of Borehole Thermal Energy Storage in Waste Heat Recovery from Diesel Generators in Remote Cold Climate Locations* [Text] / S. A. Ghoreishi-Madiseh [et al.] // *Energies*. – 2019. – Vol. 12, no. 4. – P. 656. – DOI: 10.3390/en12040656.
- [6] Rad, F. M. *An integrated model for designing a solar community heating system with borehole thermal storage* [Text] / F. M. Rad, A. S. Fung, M. A. Rosen // *Energy for Sustainable Development*. – 2017. – Vol. 36. – P. 6–15. – DOI: 10.1016/j.esd.2016.10.003.
- [7] Nilsson, E. *Borehole Thermal Energy Storage Systems for Storage of Industrial Excess Heat: Performance Evaluation and Modelling* [Text] : Licentiate Thesis / E. Nilsson, P. Rohdin. – Linköping University, Linköping, Sweden, 2020. – DOI: 10.3384/lic.diva-163789.
- [8] Wong, B. *Drake Landing Solar Community: Financial Summary and Lessons Learned* [Text] / B. Wong, L. Mesquita // *Proceedings of the ISES Solar World Congress 2019*. – 2019. – DOI: 10.18086/swc.2019.11.03.
- [9] Ayompe, L. M. *Analysis of the thermal performance of a solar water heating system with flat plate collectors in a temperate climate* [Text] / L. M. Ayompe, A. Duffy // *Applied Thermal Engineering*. – 2013. – Vol. 58, no. 1–2. – P. 447–454. – DOI: 10.1016/j.applthermaleng.2013.04.062.
- [10] Tian, Z. *Solar heating plants with flat plate collectors and parabolic trough collectors* [Text] / Z. Tian. – Technical University of Denmark, Department of Civil Engineering, 2018. – (B Y G D T U. Rapport ; No. R-394).
- [11] Behzadi, A. *Dynamic seasonal energy management of borehole thermal energy storage and smart heat pump synergies in fossil-free, ultra-efficient buildings* [Text] / A. Behzadi [et al.] // *Applied Energy*. – 2026. – Vol. 406. – P. 127261. – DOI: 10.1016/j.apenergy.2025.127261.
- [12] Xiao, H. *Application of Photovoltaic and Solar Thermal Technologies in Buildings: A Mini-Review* [Text] / H. Xiao [et al.] // *Coatings*. – 2024. – Vol. 14, no. 3. – P. 257. – DOI: 10.3390/coatings14030257.
- [13] Beaufait, R. *Techno-Economic Analysis of a Seasonal Thermal Energy Storage System with 3-Dimensional Horizontally Directed Boreholes* [Text] / R. Beaufait [et al.] // *Thermo*. – 2022. – Vol. 2, no. 4. – P. 453–481. – DOI: 10.3390/thermo2040030.
- [14] Mastný, P. *Mathematical Modeling of Basic Parts of Heating Systems with Alternative Power Sources* [Text] / P. Mastný, J. Morávek, J. Pitron // *Recent Advances in Fluid Mechanics and Thermal Engineering*. – Salerno, Italy, 2015. – P. 126–131.
- [15] Flat-plate solar collectors, SchücoSol K [Electronic resource] : product information sheet. – URL: <https://www.construction21.org/espana/data/sources/users/1651/schucosol-solar-thermal-compact-line-product-information-sheet.pdf>